



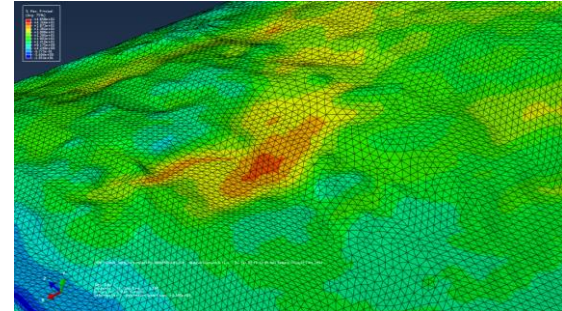
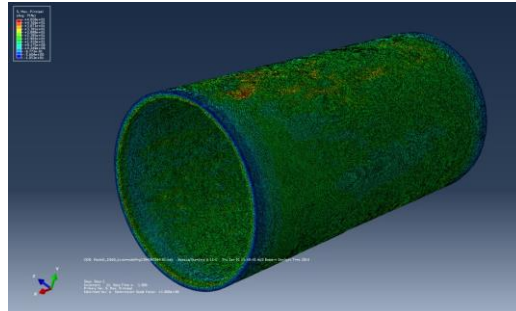
# Critical Pipe Project


## Activity 4e – Probabilistic Physical Modelling

### Progress Presentation

Prof. Jayantha Kodikara  
Dr. Jian Ji

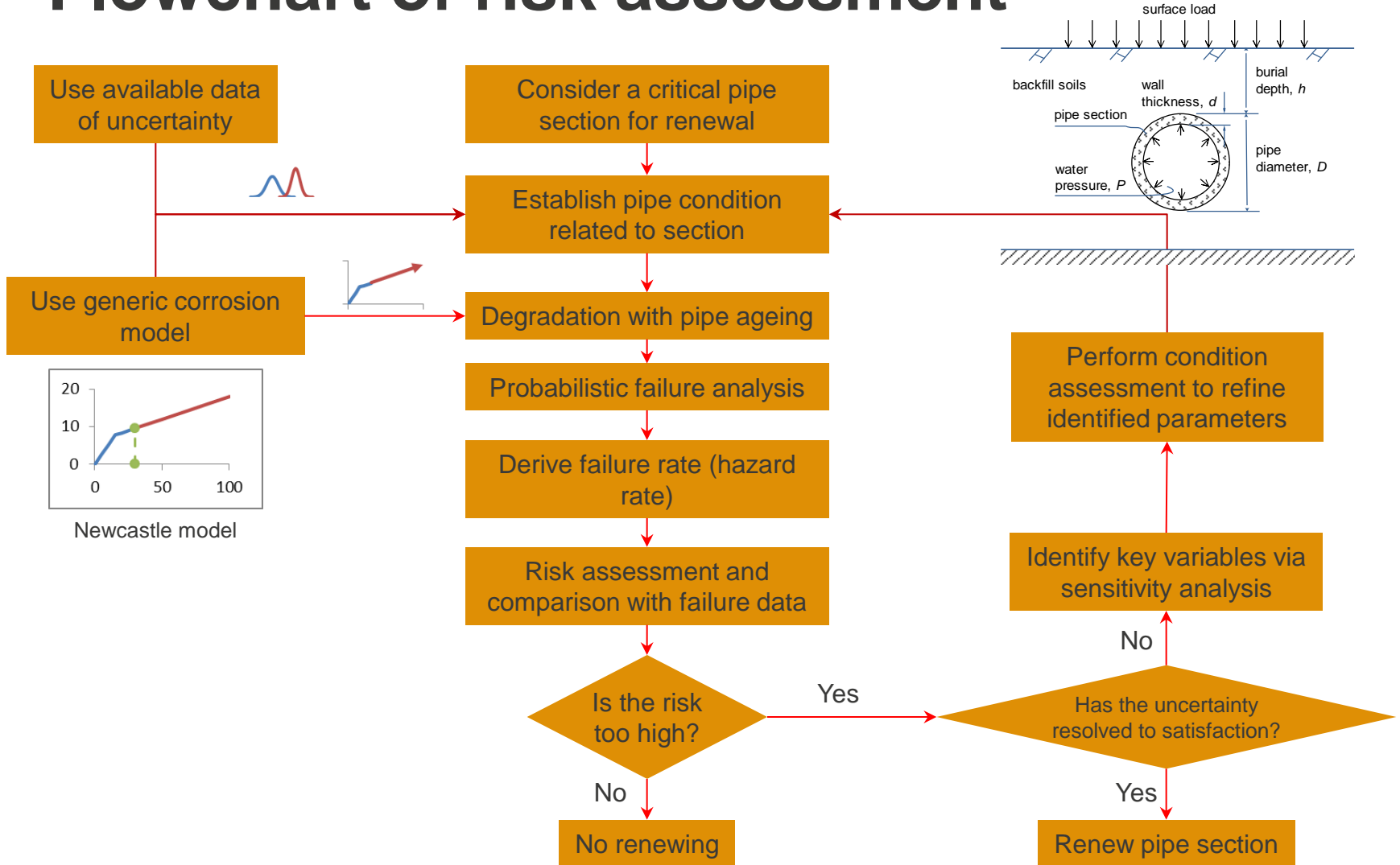
Clayton Campus  
4<sup>th</sup> Mar 2015



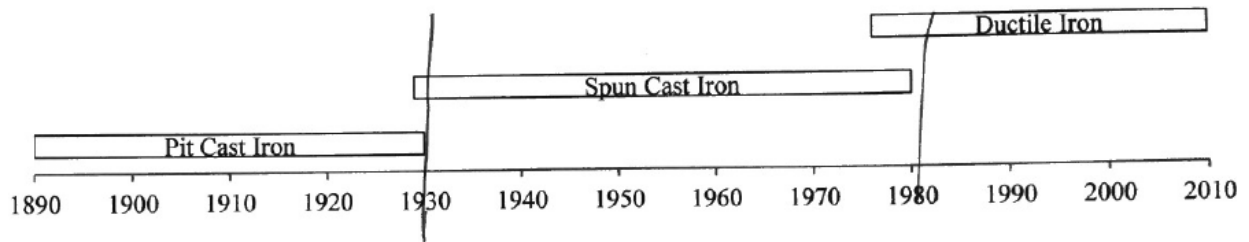


# **Probabilistic physical modelling (PPM) of cast iron pipe (bottom-up approach)**

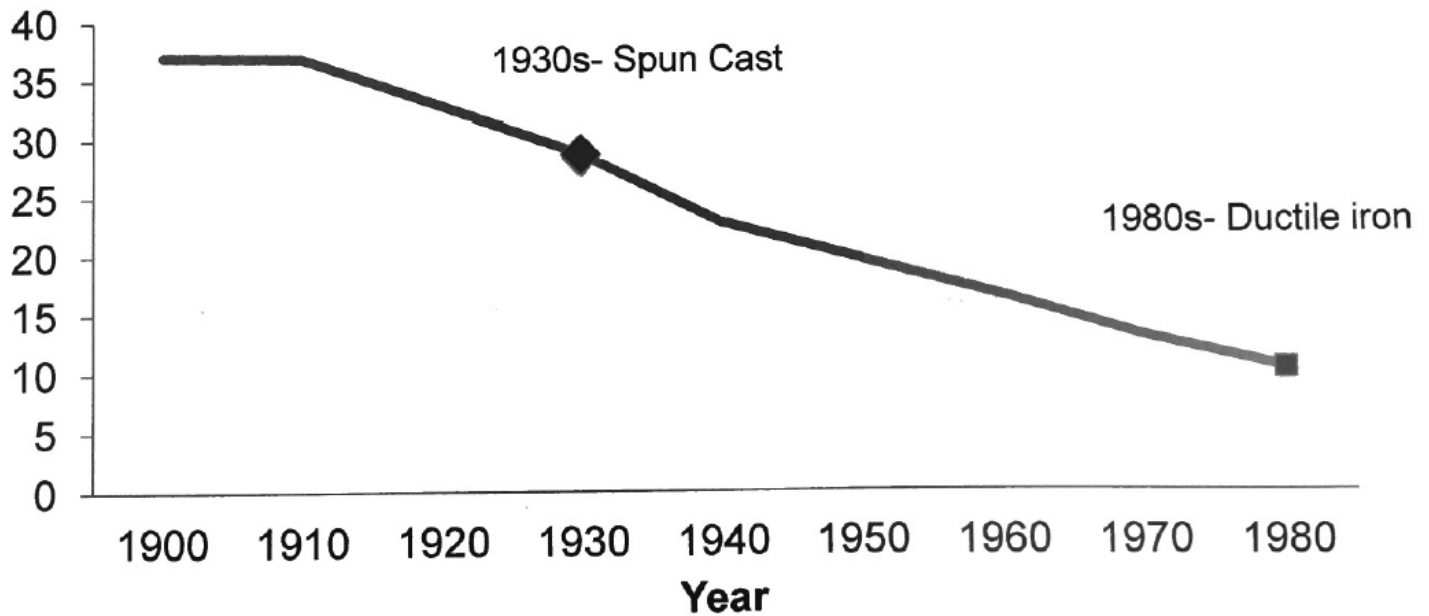
# Flowchart of risk assessment



# Literature review- dimensions of CI Pipes



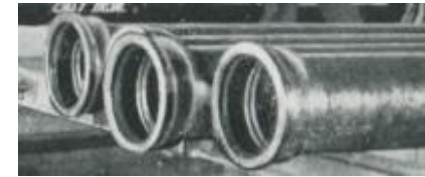
Wall thickness (mm)  
(D=600mm)



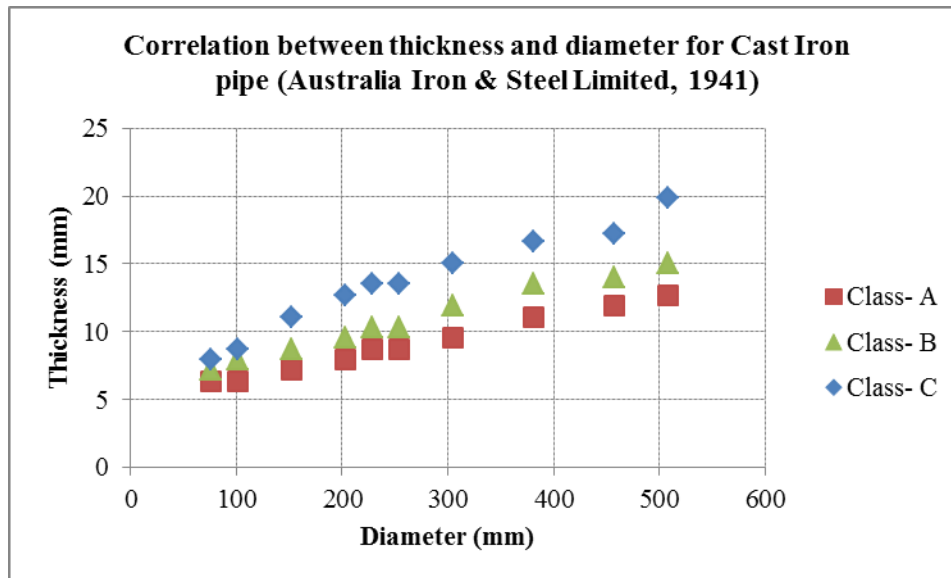
BYCL

# Correlation between pipe diameter and thickness

Historical data from manufacturer, AIS, 1941



Standard Specification for  
Super DeLavaud  
Centrifugally Cast Iron Pipes



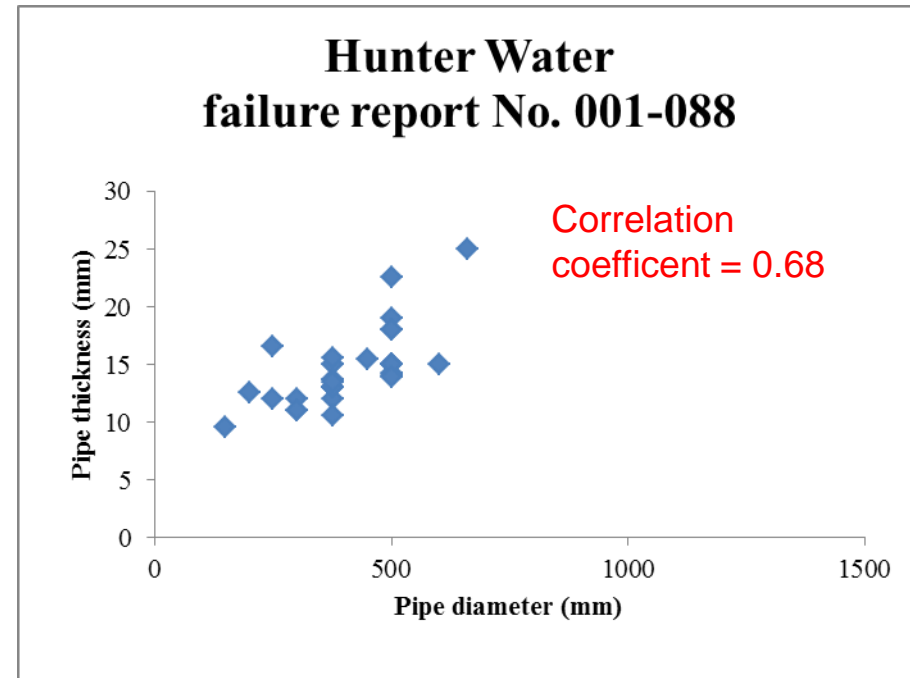
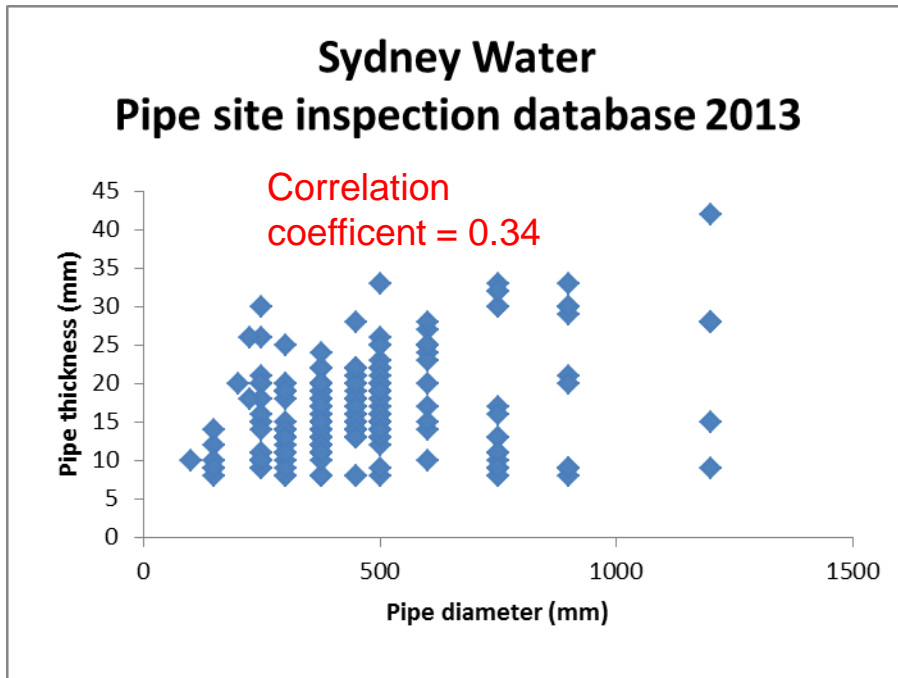
Correlation coefficient:

0.98 (Class- A)

0.99 (Class- B)

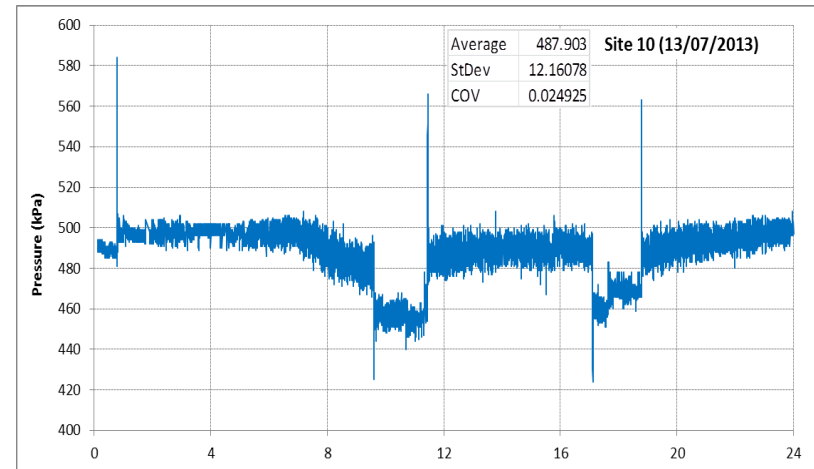
0.98 (Class- C)

Historical data from utility site inspection

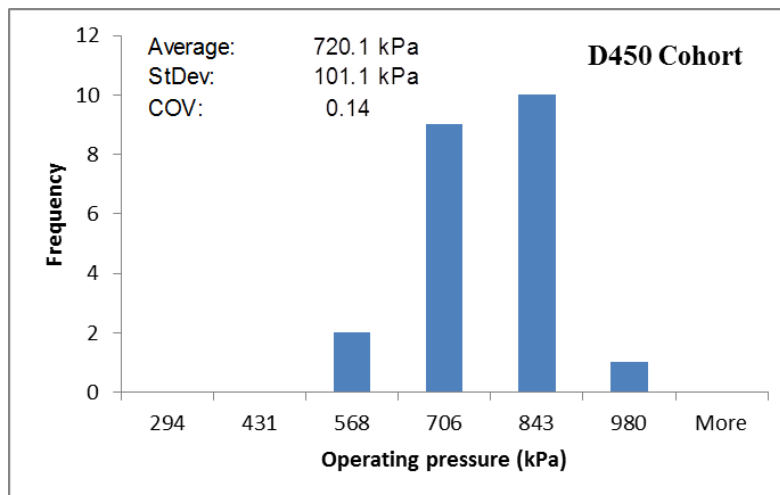
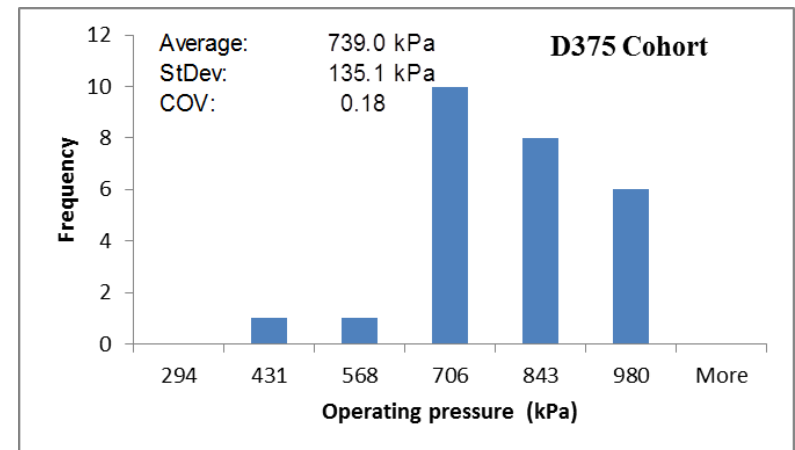
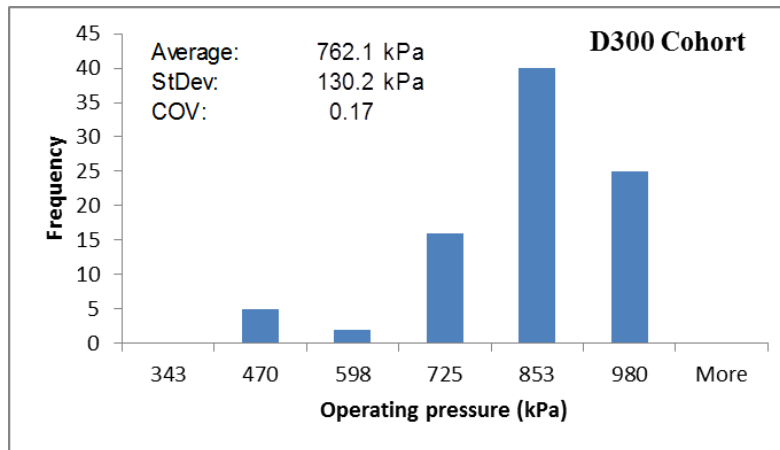


# Variation of water pressure at a testing site - Lookout long-term

1. Water pressure at a testing site consists of transient and steady-state pressures.
2. Due to large volume of steady-state record data, these peak values representing the transient could be unrealistically averaged, hence unable to characterize the variation.
3. It is more realistic to group the transient and steady-state pressure to describe the variation of water pressure



# Variation of water pr in pipe cohort of the same diameter



Reference: [City West Water data](#)

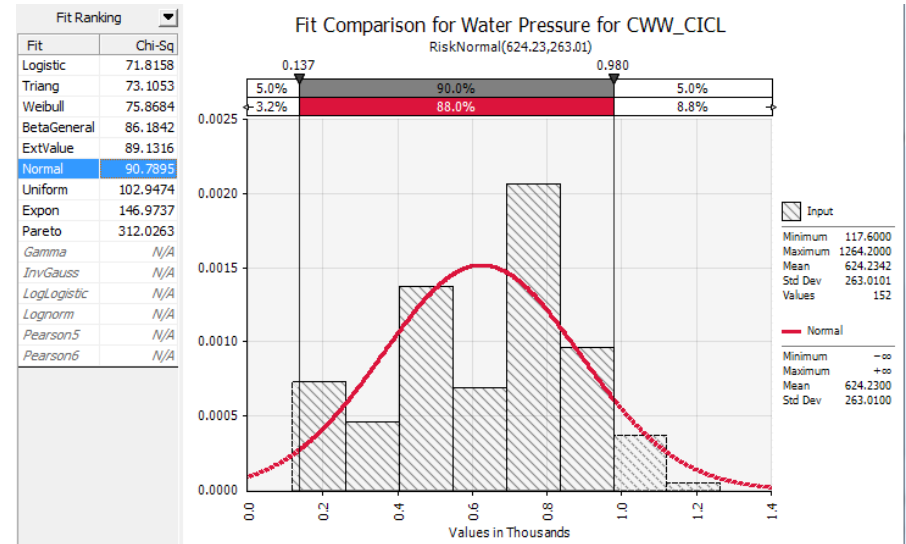
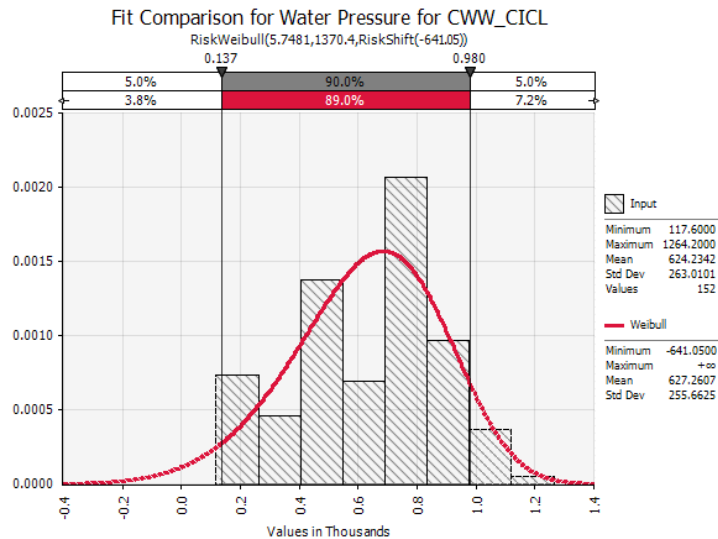
water-main failures for Monash critical data since 1997

Observation: [Water pressure for pipe cohort of the same diameter has relatively low coefficient of variation, typically varying between 0.15 to 0.20](#)

# Statistics of water pr in CICL Cohort

Observation: Water pressure for pipe cohort of the same material but mixed diameter has relatively larger coefficient of variation, typically varying up to 0.42; Several statistical distributions may be fitted to the data, including Triangular, Weibull, Normal or LogNormal. It seems there is no priority of one to others. For simplicity, Normal or LogNormal is sufficient for probabilistic physical modelling

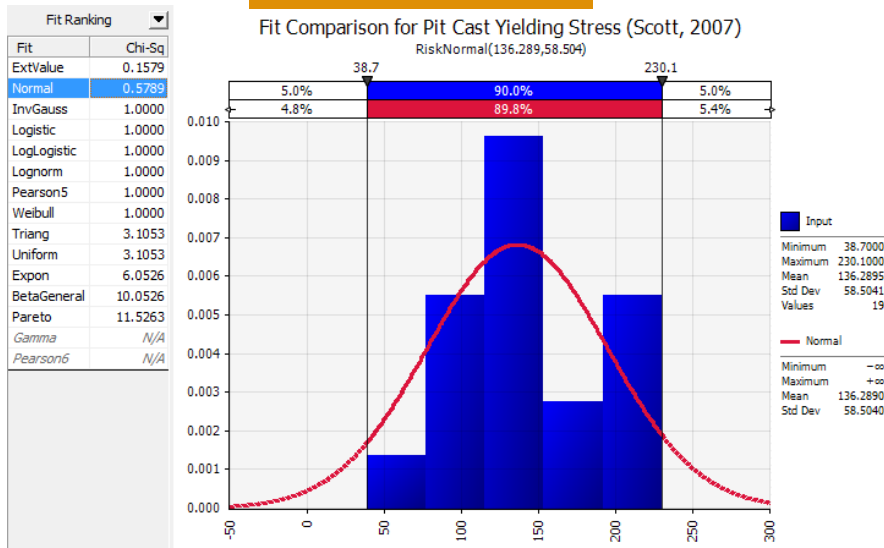
Fit	Chi-Sq
Logistic	71.8158
Triang	73.1053
Weibull	75.8684
BetaGeneral	86.1842
ExtValue	89.1316
Normal	90.7895
Uniform	102.9474
Expon	146.9737
Pareto	312.0263
Gamma	N/A
InvGauss	N/A
LogLogistic	N/A
Lognorm	N/A
Pearson5	N/A
Pearson6	N/A



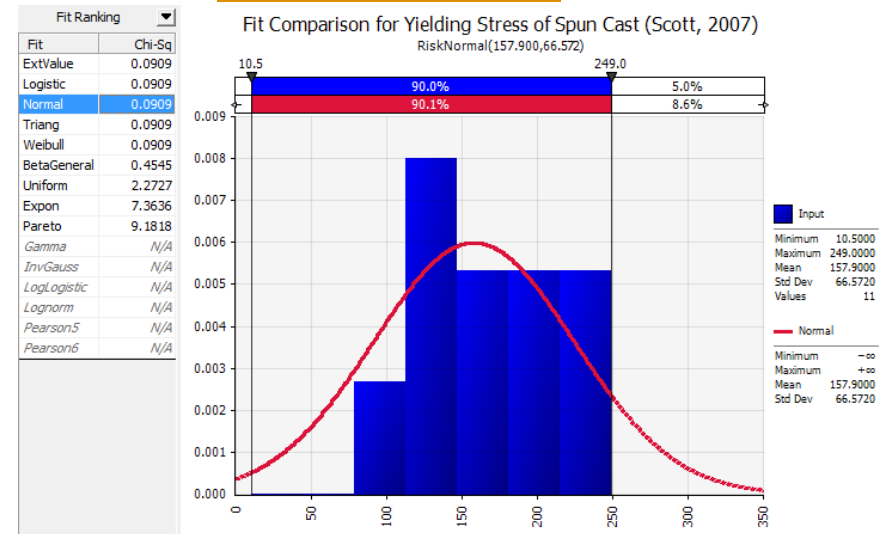
# Statistical analysis of CI yielding stress

- Statistics of yielding stress for pipe cohorts

## Pit cast iron



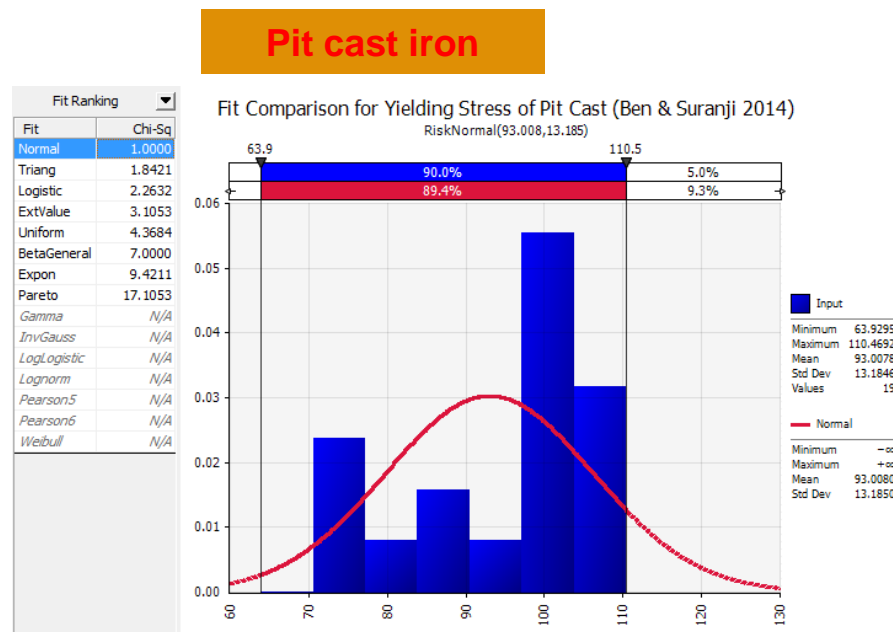
## Spun cast iron



- On average, the yielding stresses are 136 kPa for pit cast, and 158 kPa for spun cast iron. The percentage difference is 16 %. Both pit cast and spun cast iron pipes are subjected to high coefficient of variation (c.o.v.) of value up to 40%

# Statistical analysis of CI yielding stress

- Statistics of yielding stress for an **isolated pipe**



Conclusion:

When analyzing a single pipe failure, use small c.o.v. ranged 0.1 to 0.2

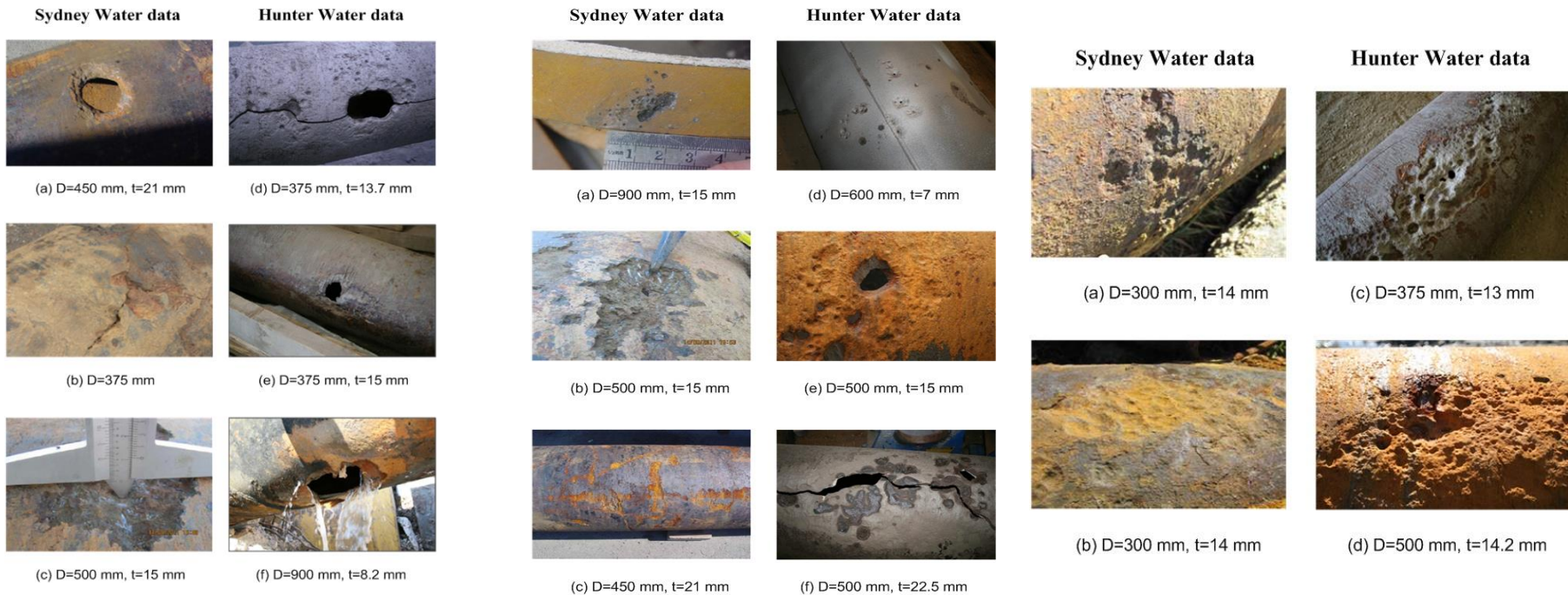
When analyzing pipe cohort, use a bigger c.o.v. ranged 0.3 to 0.5

- On average, the yielding stresses is 93 kPa for pit cast. The coefficient of variation of value is around 15%, which is significantly less than the pipe cohort.

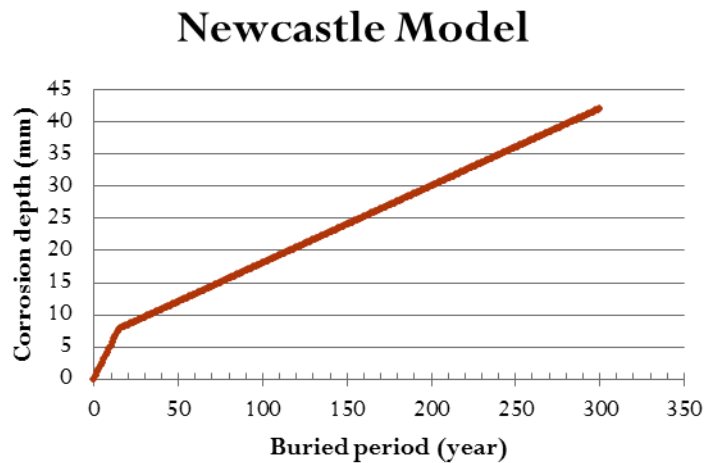
# Pipe stress prediction on pitting corrosion pipes

## Type of corrosion

- **Pitting corrosion (Type – III)** refers to localised regions of metal loss that can be characterised by pit geometry.



# Corrosion pit depth v.s. pipe ageing

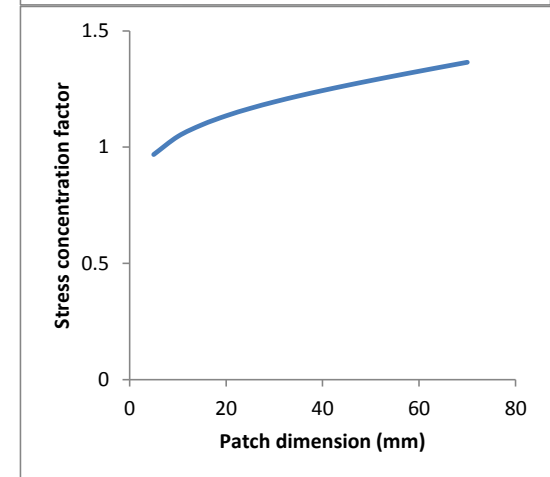
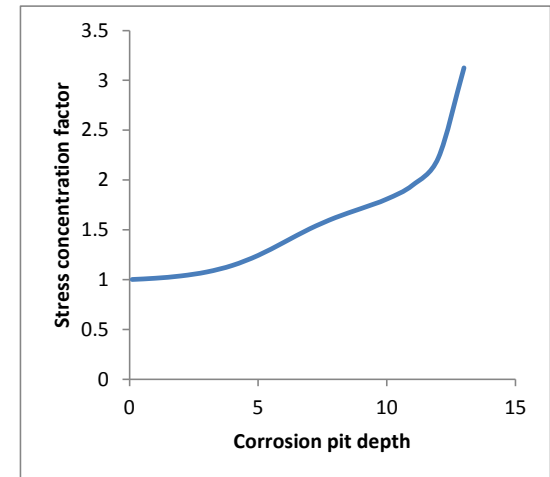
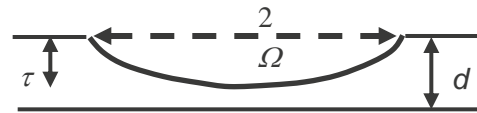
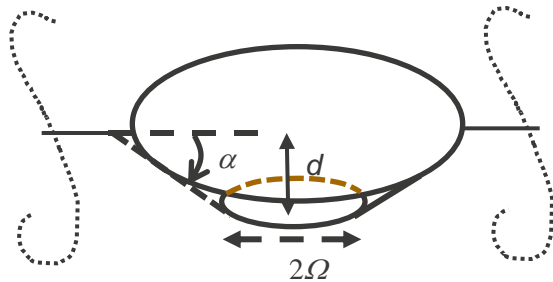


$$\begin{cases} \tau(t) = c_s + r_s \times t, & \text{when } t \geq T^* \\ \tau(t) = (c_s / T^* + r_s) \times t, & \text{when } t < T^* \end{cases}$$

Assume	Cs=	6mm
	rs=	0.12
	T*=	15

# Effect of Patch Corrosion in Pipe Stress

## Non-Linear regression model for stress concentration factor (circular)



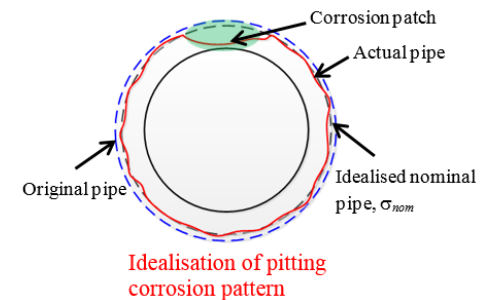
$$SCF_{TWC} = 5.4998 \times \left\{ \frac{\sqrt[4]{3(1-\nu^2)}}{2} \cdot \left( \frac{\Omega}{\sqrt{Rd}} \right)^{1.2} \cdot \left( \frac{1}{\sin \alpha} \right)^{1.5} \right\}^{0.3377}$$

$$SCF_{RWC} = \frac{\left\{ 1.0 - \alpha_1 \left[ 1.0 - \left( \frac{d'}{d} \right)^{\alpha_2} \right] \cdot M(\Omega, d, R, d', \nu) \right\}}{\left\{ 1.0 - \alpha_1 \left[ 1.0 - \left( \frac{d'}{d} \right)^{\alpha_2} \right] \right\}}$$

$$\sigma(\mathbf{x}, t) = \sigma_{nom} \times SCF(t)$$

# Coupling the pitting corrosion to uniform wall thickness loss

- Buried water mains are usually subjected to complex corrosion patterns. Typically, there could be some critical pits scattered on a uniformly corroded pipe wall. This combined corrosion patterns need to be properly considered in the pipe failure mechanism.
- The following assumptions apply in this probabilistic physical modelling:
  - 1) The uniform loss of wall thickness is reasonably assumed to be 1/5 of the **most critical pit depth**.
  - 2) The isolated corrosion pit is circular from top-view
  - 3) The pit dimensions (radius) will linearly increase with pit depth



# Typical statistical information of physical properties in pipeline analysis

Physical parameter		Mean value	COV (Others)	Distribution	Source of data
Location	Bury depth, h	0.8 m	0.25	Normal	ACA&PFPP
Backfill soil surrounding pipelines	Elastic modulus, Es	25 MPa	0.3	LogNormal	Common assumptions
	Unit weight	20 kN/m <sup>3</sup>	0.1	LogNormal	
Pipe physical properties (cast iron)	Elastic modulus	100 GPa	0.05	LogNormal	Common assumptions
	Yielding stress	100 MPa	0.1	Normal	Seica and Packer (2004), ACA&PFPP
	wall thickness	12 to 35		Normal	AIS (1941), ACA&PFPP
	Pipe diameter	Varying		Normal	
Load	Surface load (traffic)	50 kN	0.3	Normal	ACA&PFPP
	Operating water pressure	1000 kPa	0.15	Normal	ACA&PFPP
Corrosion	cs	6 mm	0.3	LogNormal	Newcastle
	rs	0.12 mm/y	0.3	LogNormal	
	T*	15 y	0.01	Normal	
Pit size	$\lambda_{-u1}$	5*pit_depth mm	0.5	Normal	Typical values inferred from Hunter Water pipeline failure report, ACA&PFPP
	$\lambda_{-u1}$	5*pit_depth mm	0.5	Normal	
	$\lambda_{-b1}$	4*pit_depth mm	0.5	Normal	
	$\lambda_{-b1}$	4*pit_depth mm	0.5	Normal	

# Probabilistic physical modelling of pitting corrosion pipe

- Stress prediction of cast iron pipe by finite element analysis
- Stress concentration factor of corrosion pit
- Bilinear corrosion model from activity 3
- Time-dependent limit state function:

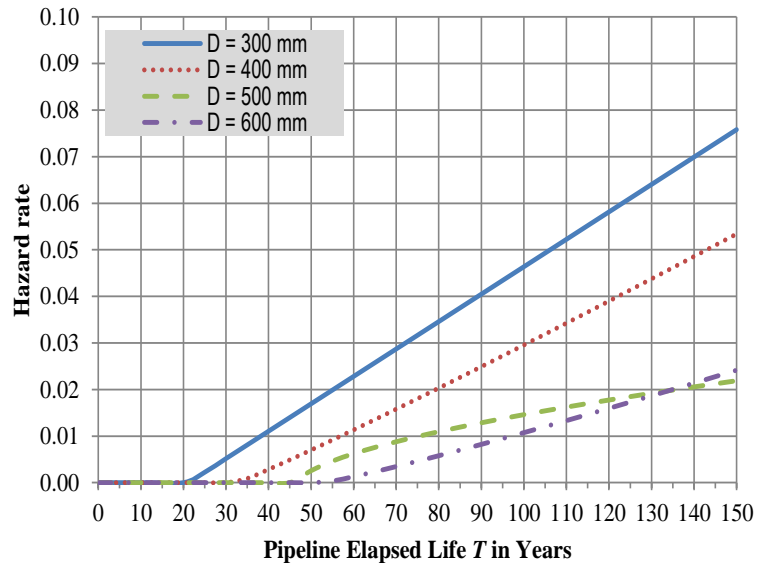
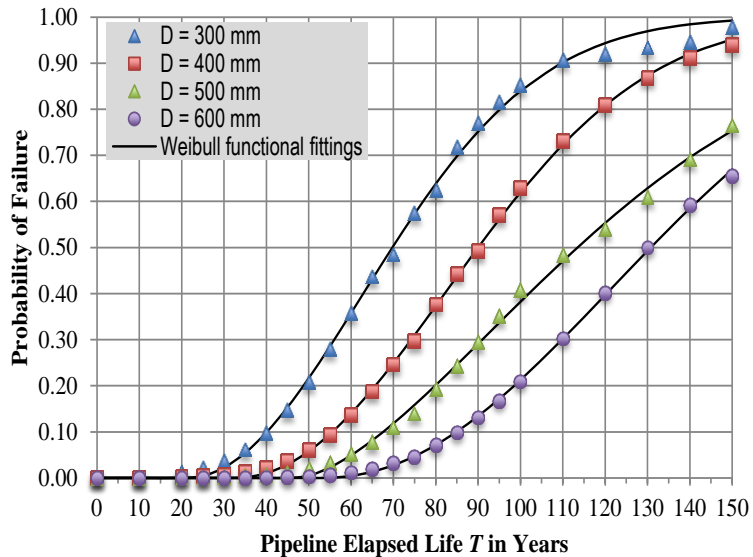
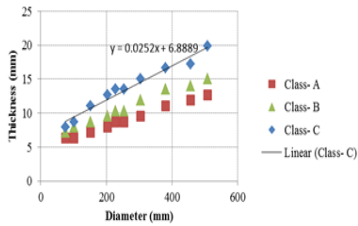
$$g(\mathbf{x}, t) = \sigma_y - \sigma_{nom}(\mathbf{x}, t) \cdot SCF(t)$$

- Probability of pipe failure:

$$P\{g(\mathbf{x}, t) < 0\} = P\{\sigma_y - \sigma_{nom}(\mathbf{x}, t) \cdot SCF(t)\}$$

# Weibull parameters inferred by PPM

Correlation between thickness and diameter for Cast Iron pipe (Australia Iron & Steel Limited, 1941)



# Three parameter Weibull functions

Failure functions (= 1- Reliability Function) with respect to failure age/time,  $T$

$$F(T : \alpha, \beta) = 1 - e^{-\{(T-T_0)/\beta\}^\alpha}$$

Hazard rate function

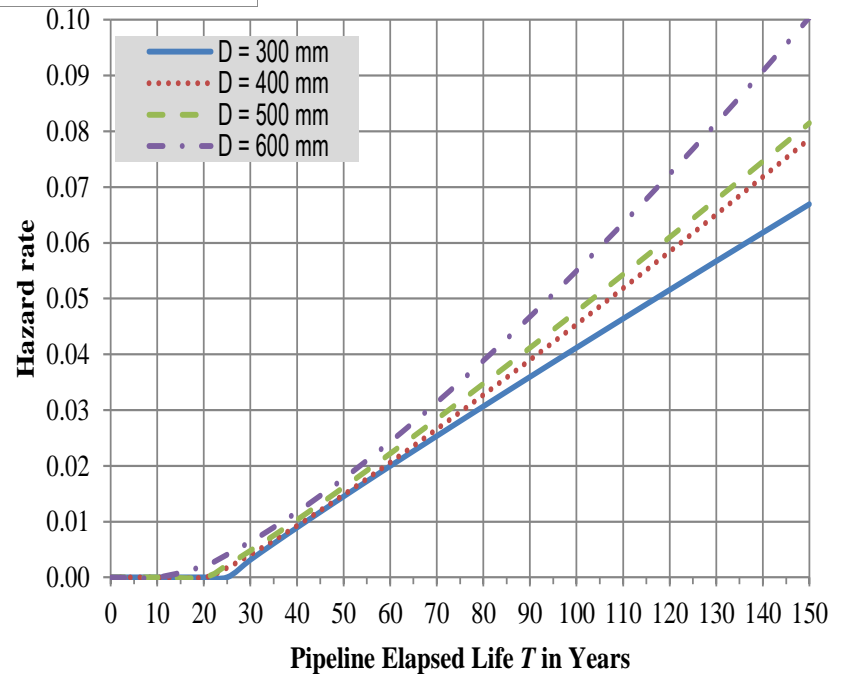
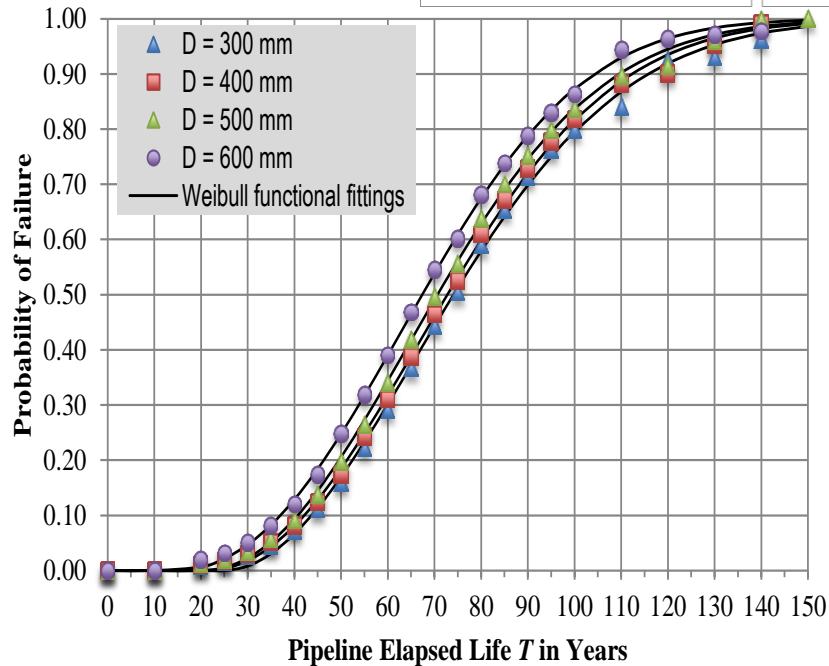
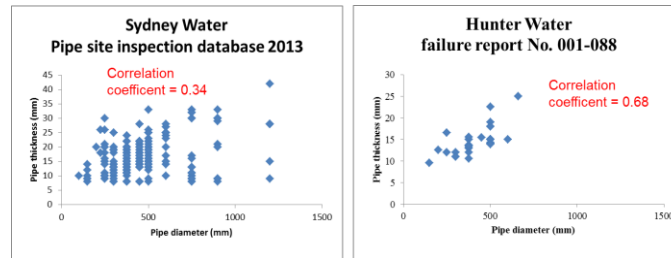
$$H(T : \alpha, \beta) = \frac{dF(T : \alpha, \beta) / dT}{1 - F(T : \alpha, \beta)} = \frac{\alpha}{\beta} \left( \frac{T - T_0}{\beta} \right)^{\alpha-1} e^{-\{(T-T_0)/\beta\}^\alpha}$$

where  $T_0$  is the location parameter,  $\alpha$  the shape parameter and  $\beta$  the scale parameter.

## Weibull parameters inferred from probabilistic physical modelling when wall thickness increases with pipe diameter

Corroded pipe cohorts	Weibull parameters			R <sup>2</sup>
	$\alpha$	$\beta$	T <sub>0</sub>	
300 mm pipe	2.00	58.33	21.19	0.998
400 mm pipe	2.07	68.72	32.44	0.999
500 mm pipe	1.61	83.57	46.94	0.998
600 mm pipe	2.12	92.69	52.78	0.999

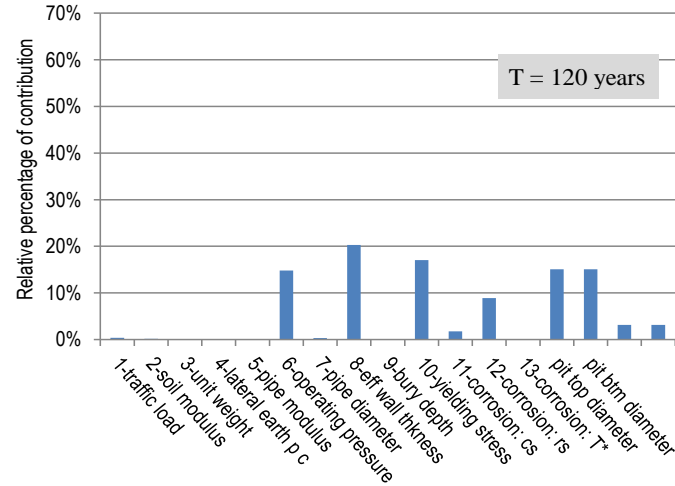
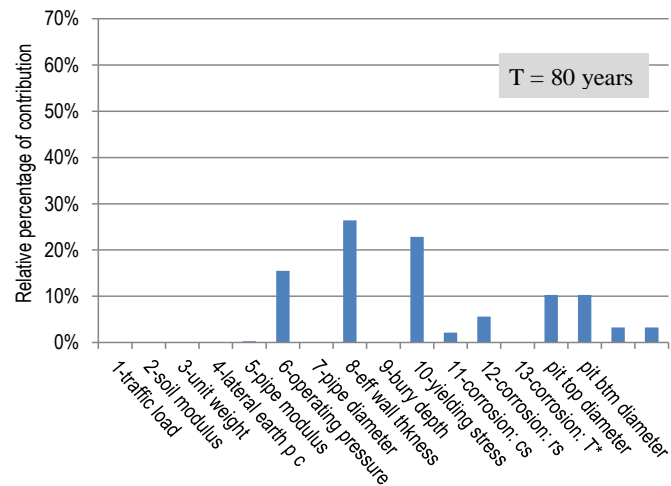
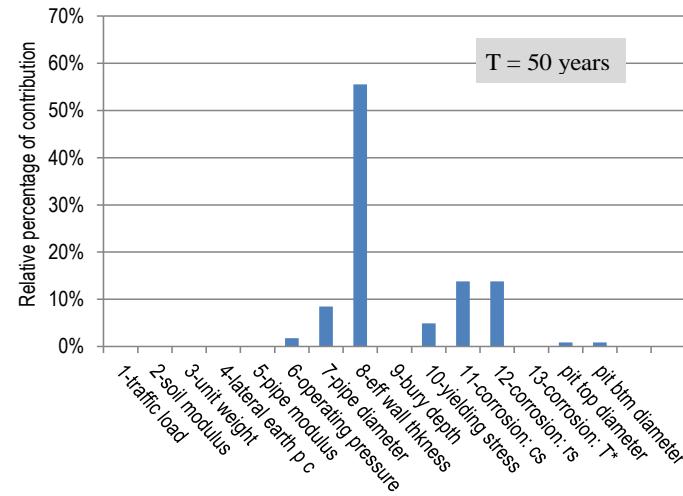
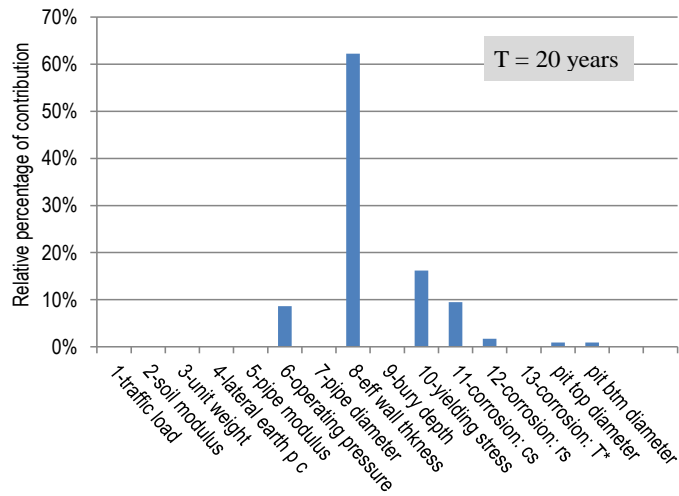
# Weibull parameters inferred by PPM



## Weibull parameters inferred from probabilistic physical modelling when wall thickness is fixed to be 15 mm

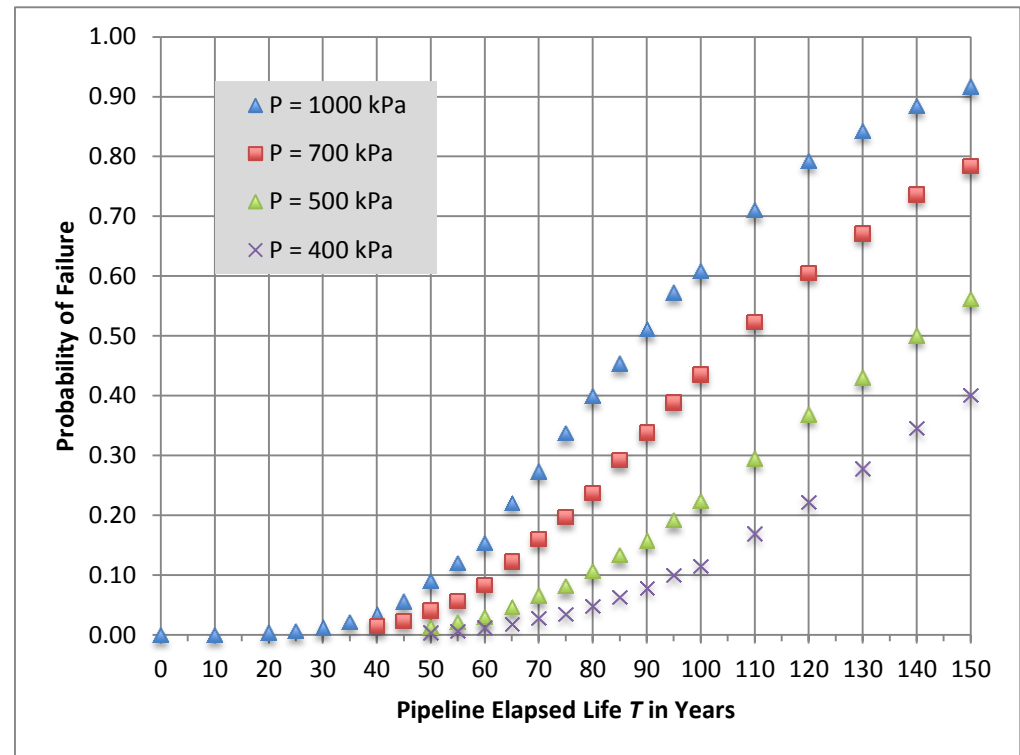
Corroded pipe cohorts	Weibull parameters			Fitting index $R^2$
	$\alpha$	$\beta$	$T_0$	
300 mm pipe	1.95	59.26	24.99	0.999
400 mm pipe	2.13	61.92	20.66	0.995
500 mm pipe	2.10	60.22	20.07	0.992
600 mm pipe	2.33	64.17	12.55	0.999

# Change of parameter sensitivity with pipe ageing



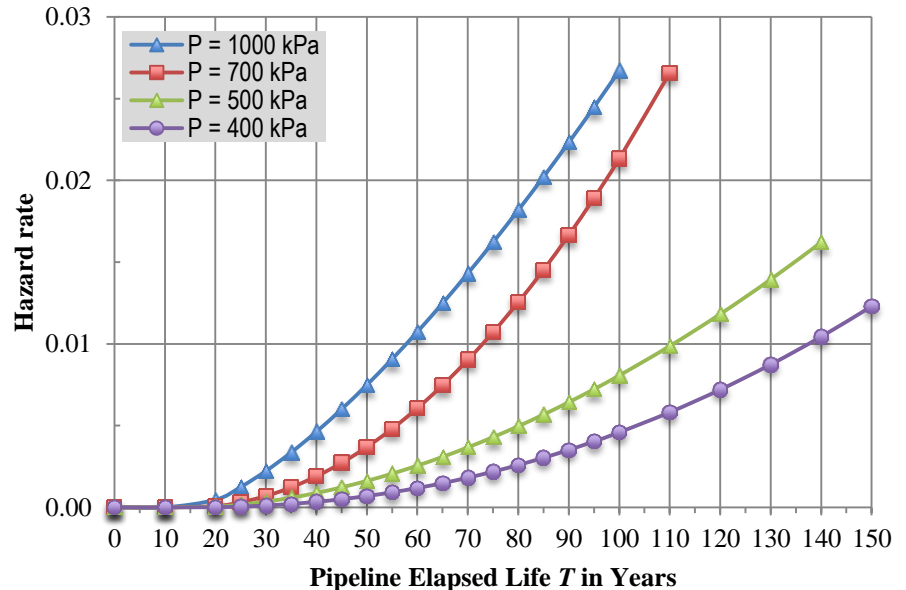
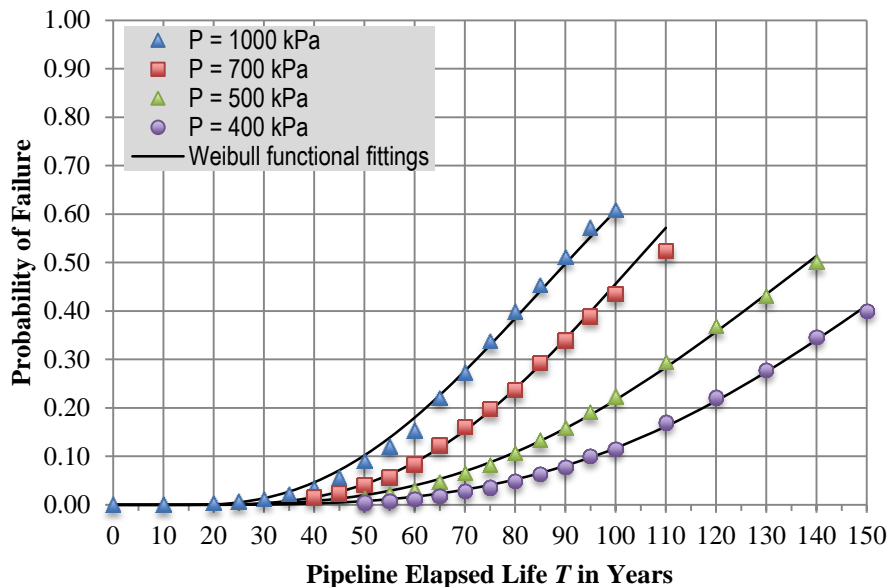
# Parametric study: operating pressure v.s. failure probability

	Symbol	Description	Mean	COV
Physical properties	$W$	1-traffic load	50 kN	0.3
	$E_s$	2-soil modulus	2.50E+04 Kpa	0.3
	$\gamma$	3-unit weight	20 kN/m3	0.1
	$k$	4-lateral earth p c	0.4	0.15
	$E_p$	5-pipe modulus	1.00E+08 Kpa	0.03
	$P$	6-operating pressu	4.00E+02 Kpa	0.15
	$D$	7-pipe diameter	0.400 m	0.05
	$t$	8-eff wall thkness	13.97 mm	0.05
	$h$	9-bury depth	0.8 m	0.25
	$s_y$	10-tension strength	1.00E+05 Kpa	0.1
corrosion	$cs$	11-corrosion: cs	6 mm	0.3
	$rs$	12-corrosion: rs	0.12 mm/y	0.3
	$T^*$	13-corrosion: T*	15 year	0.1
pit geometry	$\Omega_{U1}$	pit top diameter	75.00 mm	0.5
	$\Omega_{U2}$	pit top diameter	75.00 mm	0.5
	$\Omega_{B1}$	pit btm diameter	60.00 mm	0.5
	$\Omega_{B1}$	pit btm diameter	60.00 mm	0.5



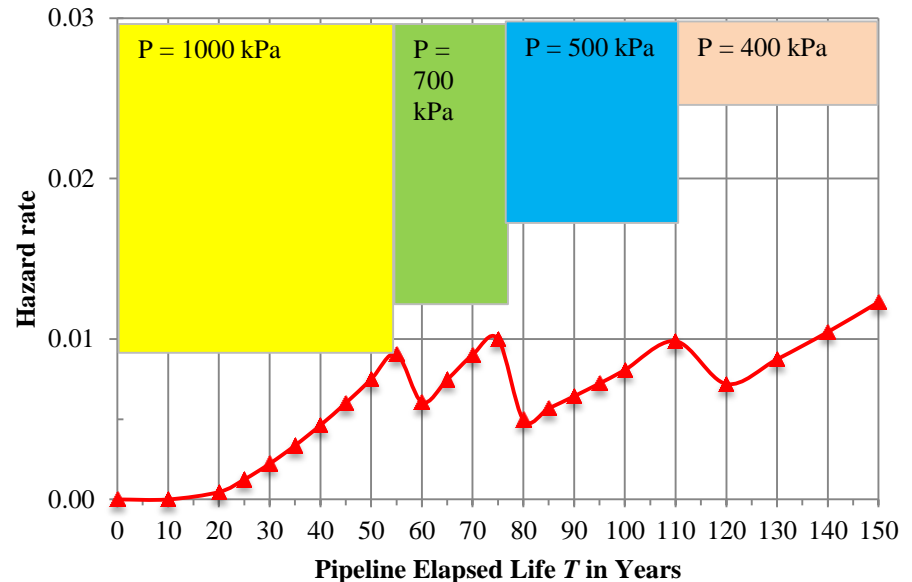
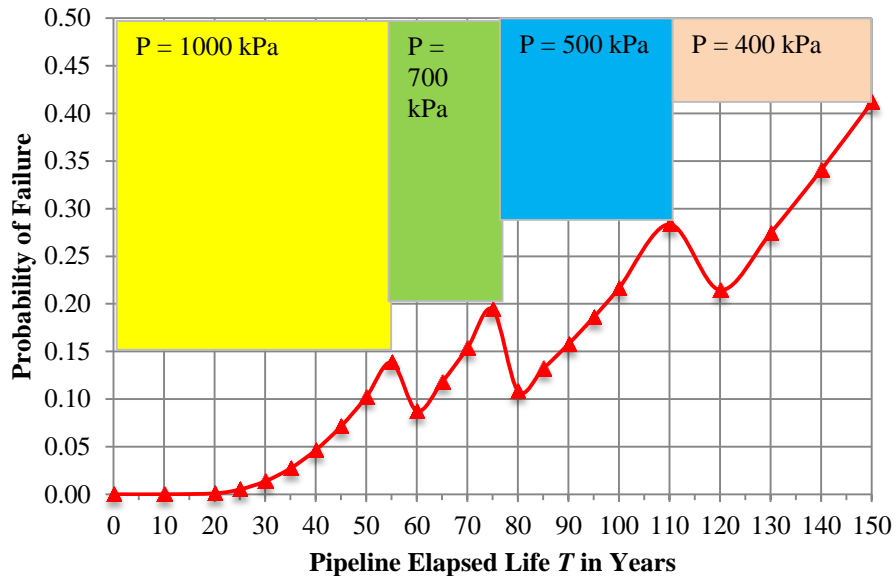
# Indications

- The probability of failure resulted from physical modelling can be well fitted by Weibull functional curves
- It is possible to derive hazard rates from Weibull functional fittings
- Hazard rate indicates the conditional probability of failure in next year

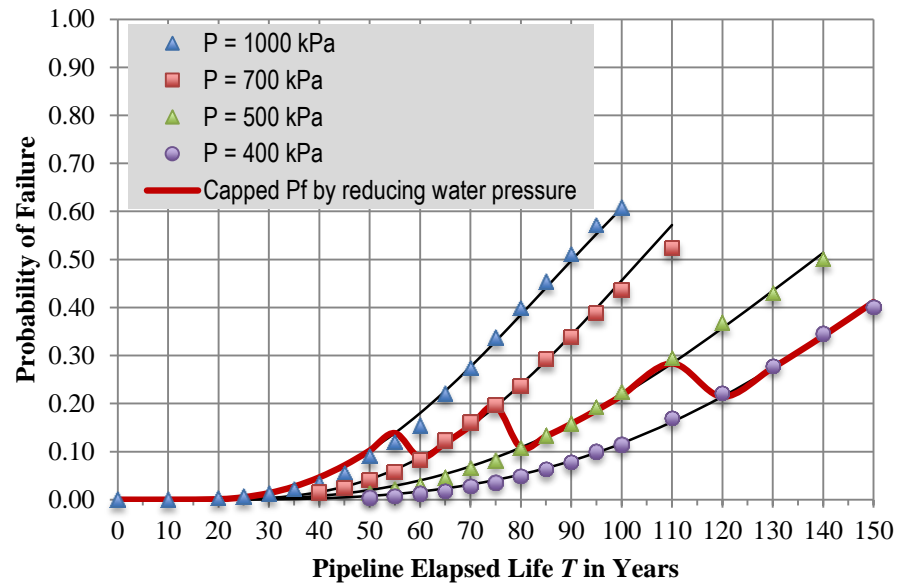


# Indications

- Maintain the hazard rate and/or failure rate, for example, probability of failure in next year not exceeding 1%, by adjusting operating pressure over servicing time

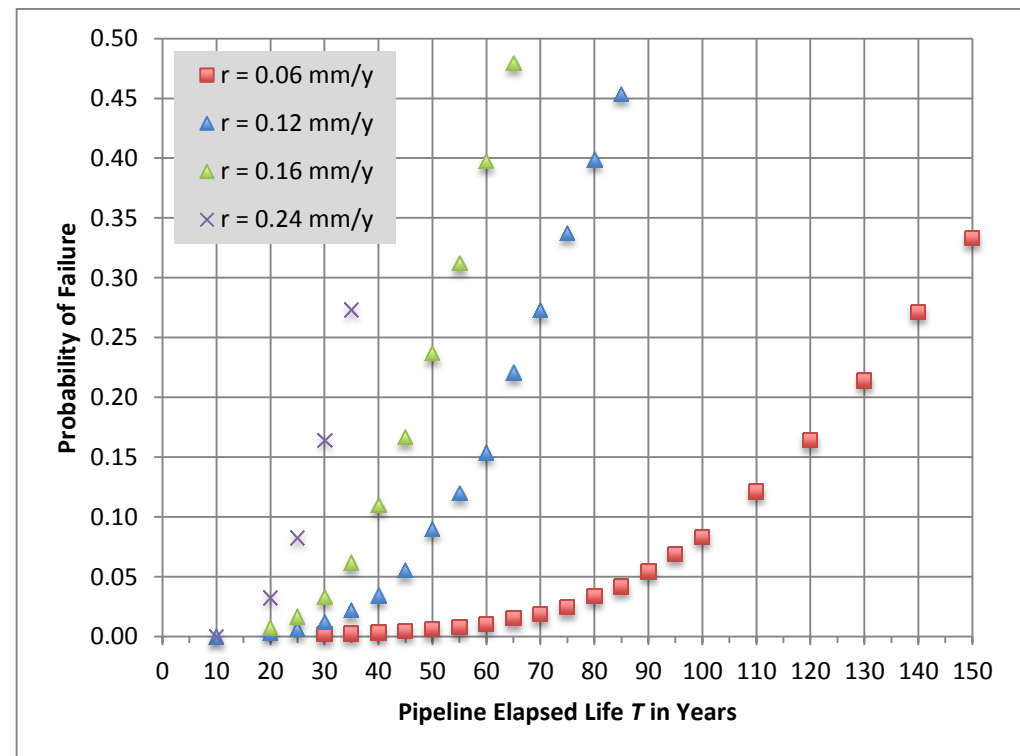


- Superimposition of  $P_f$  curves from various case studies of water pressure

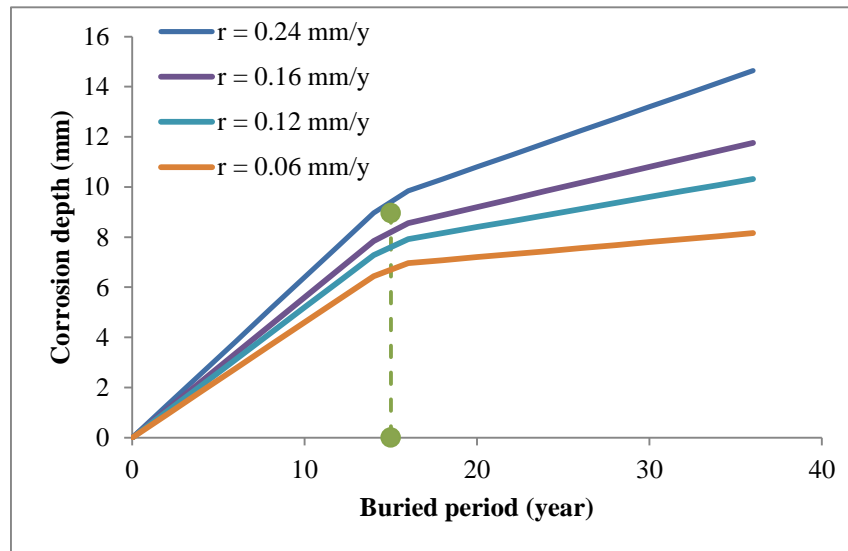


# Parametric study: corrosion rate (mm/y) v.s. failure probability

	Symbol	Description	Mean	COV
Physical properties	$W$	1-traffic load	50 kN	0.3
	$E_s$	2-soil modulus	2.50E+04 Kpa	0.3
	$\gamma$	3-unit weight	20 kN/m <sup>3</sup>	0.1
	$k$	4-lateral earth p c	0.4	0.15
	$E_p$	5-pipe modulus	1.00E+08 Kpa	0.03
	$P$	6-operating pressu	1.00E+03 Kpa	0.15
	$D$	7-pipe diameter	0.400 m	0.05
	$t$	8-eff wall thknss	14.57 mm	0.05
	$h$	9-bury depth	0.8 m	0.25
	$s_y$	10-tension strength	1.00E+05 Kpa	0.1
corrosion	$cs$	11-corrosion: cs	6 mm	0.3
	$rs$	12-corrosion: rs	0.12 mm/y	0.3
	$T^*$	13-corrosion: T*	15 year	0.1
pit geometry	$\Omega_{U1}$	pit top diameter	60.00 mm	0.5
	$\Omega_{U2}$	pit top diameter	60.00 mm	0.5
	$\Omega_{B1}$	pit btm diameter	48.00 mm	0.5
	$\Omega_{B1}$	pit btm diameter	48.00 mm	0.5

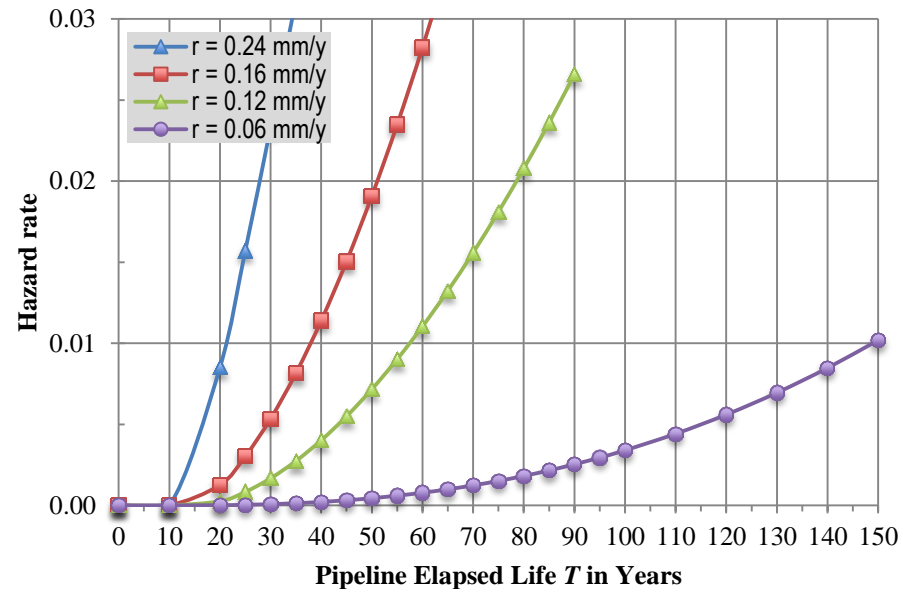
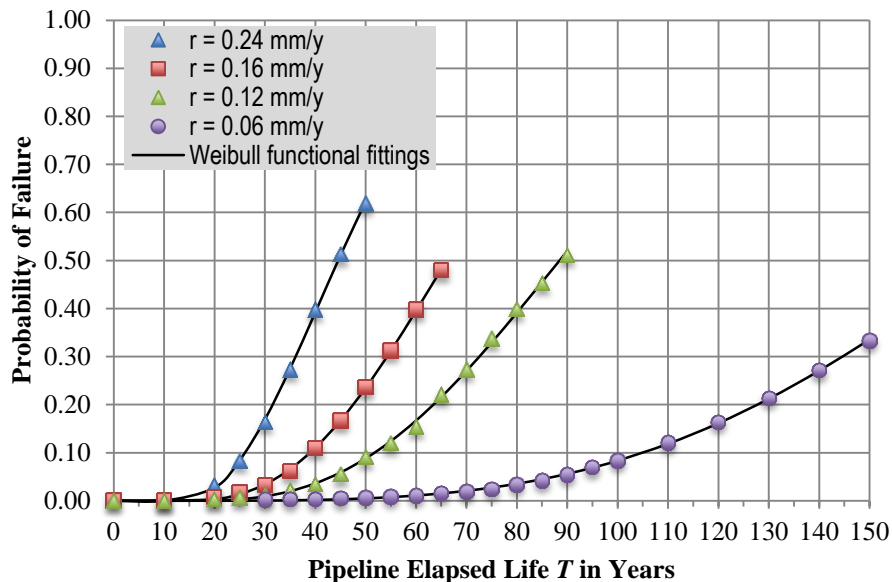
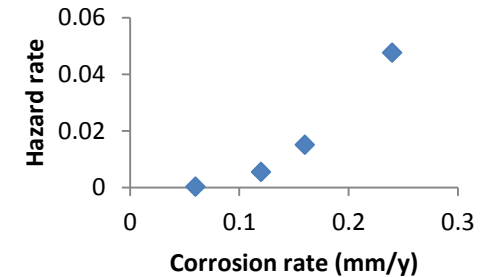


- Bilinear corrosion model



# Findings

- Corrosion rate is a key parameter influencing the failure probability
- Over time, failure rate increases almost exponentially with corrosion rate



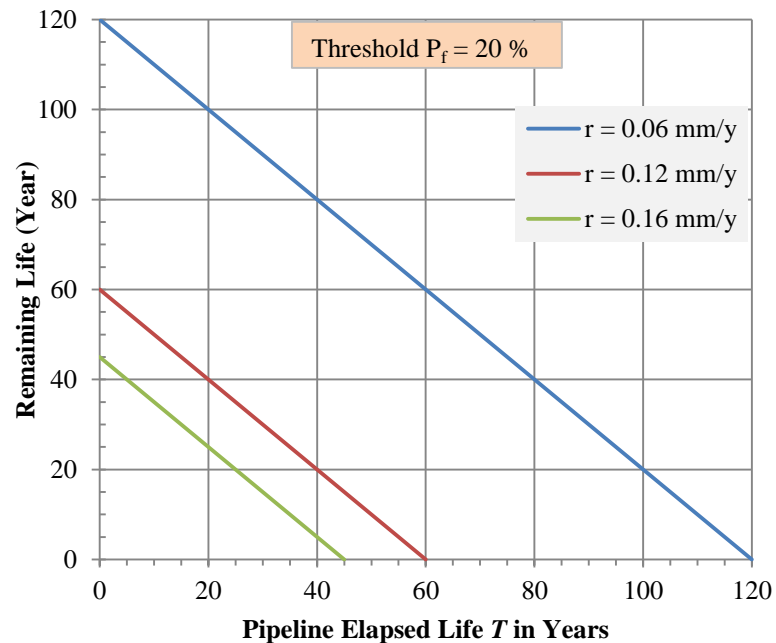
# Prediction of pipe remaining life

- Concept of risk-based remaining life prediction
  - Risk = Pf x (Consequence of failure)
  - Depending of pipe and location, there will be some consequence of a pipe failure, for example, a consequence = \$\$\$ 500,000.00
  - If we like to tolerate a risk at \$\$\$ 100,000.00, then the threshold of tolerable failure probability is

$$Pf \text{ (threshold)} = 100/500 = 20 \%$$

# Prediction of pipe remaining life

- Remaining life chart w.r.t three scenarios of corrosion rate



Note: operating pressure  $P = 1000$  kPa



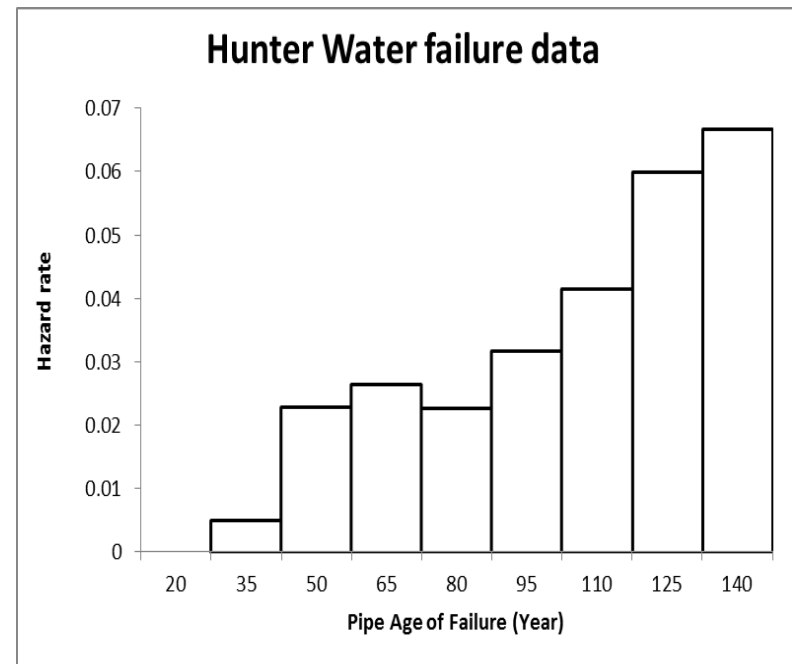
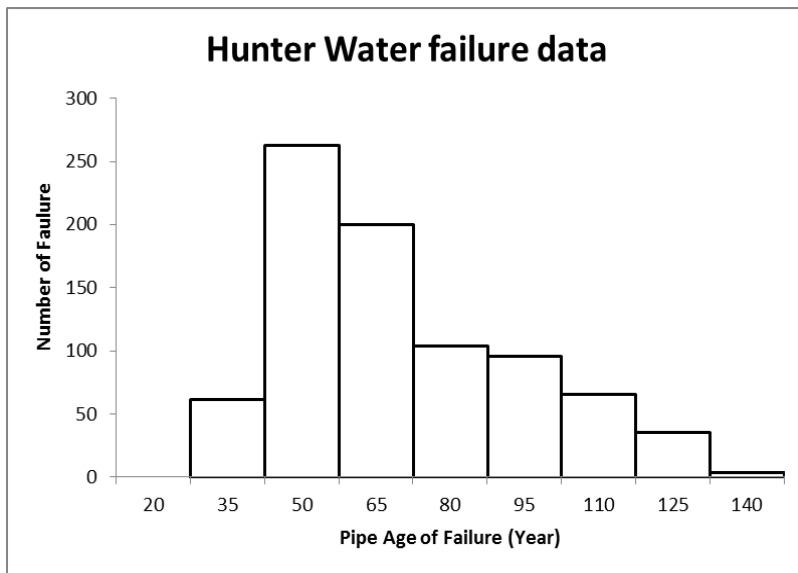
# **Statistical analysis of utility failure data (top-down)**

# Principle

- The basis of top-down assessment of pipelines is the gathering of failure data. According to Hunter Water, there are 831 cast iron cement lined (CICL) water main failures due to various reasons, as recorded from 1998 to 2012. Similarly, there are 1556 CICL water main failures recorded by Sydney Water from 2000 to 2012. These broken pipes were installed in last century ranging from 1900 to 1978.
- By sorting the observed data by pipe failure age, the probability mass function, PMF, against time is first explored. For the convenience of mathematical comparison, the so obtained PMF is transformed to hazard rate which is well-known in studying the time-to-failure systems.
- The comparison here is made between the observed failure data and the probabilistic physical modelling with a mean value of pipe wall thickness of 15 mm subjected to pitting corrosions

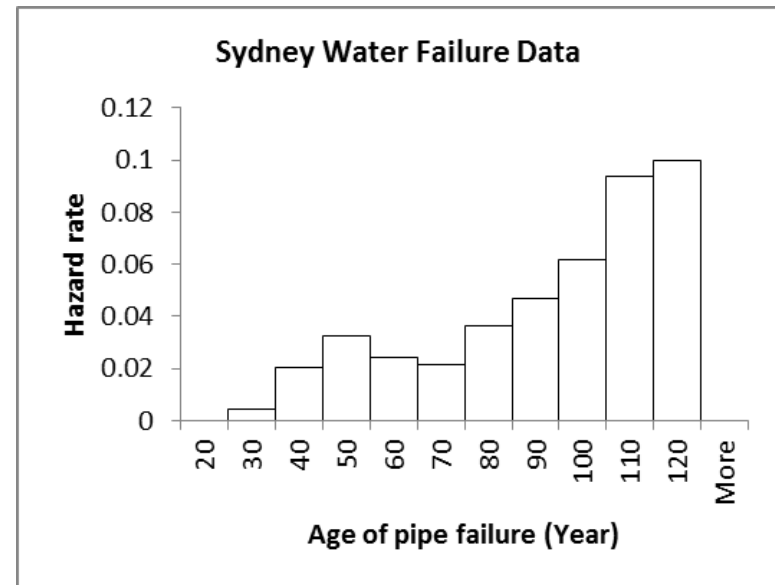
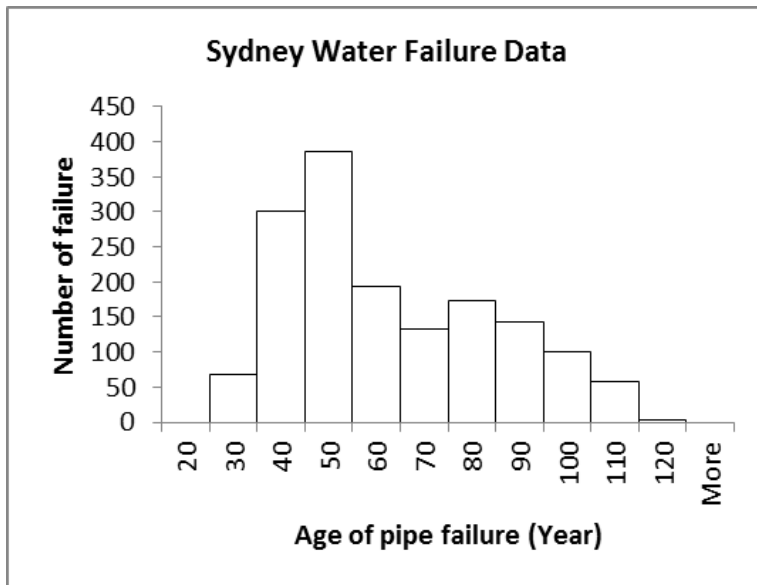
# Hunter Water: 831 CICL failures between 1998-2012

Note: The number of classes can be roughly determined by Sturges' rule:  $k = 1 + 3.3 \log_{10} N$



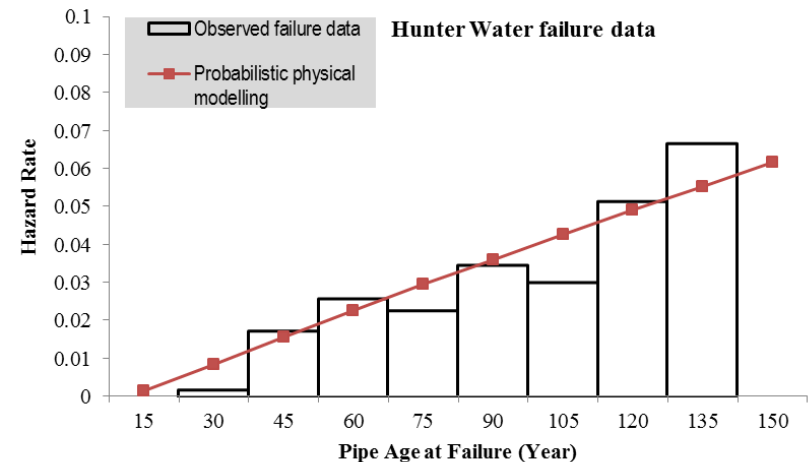
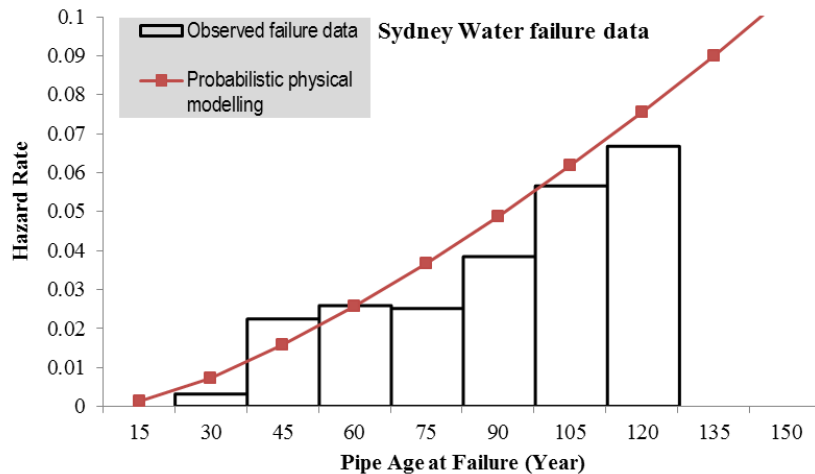
# Sydney Water: 1556 CICL failures between 2000-2012

Sturges' rule:  $k = 1 + 3.3 \log_{10} N$



# Comparison between bottom-up and top-down analyses in terms of hazard rate

- Hazard rate, which is a commonly referred parameter in time-dependent reliability engineering analysis, is adopted here for comparing the top-down and bottom-up results for critical pipe failure study.



# Estimating the number of failures/y/100km

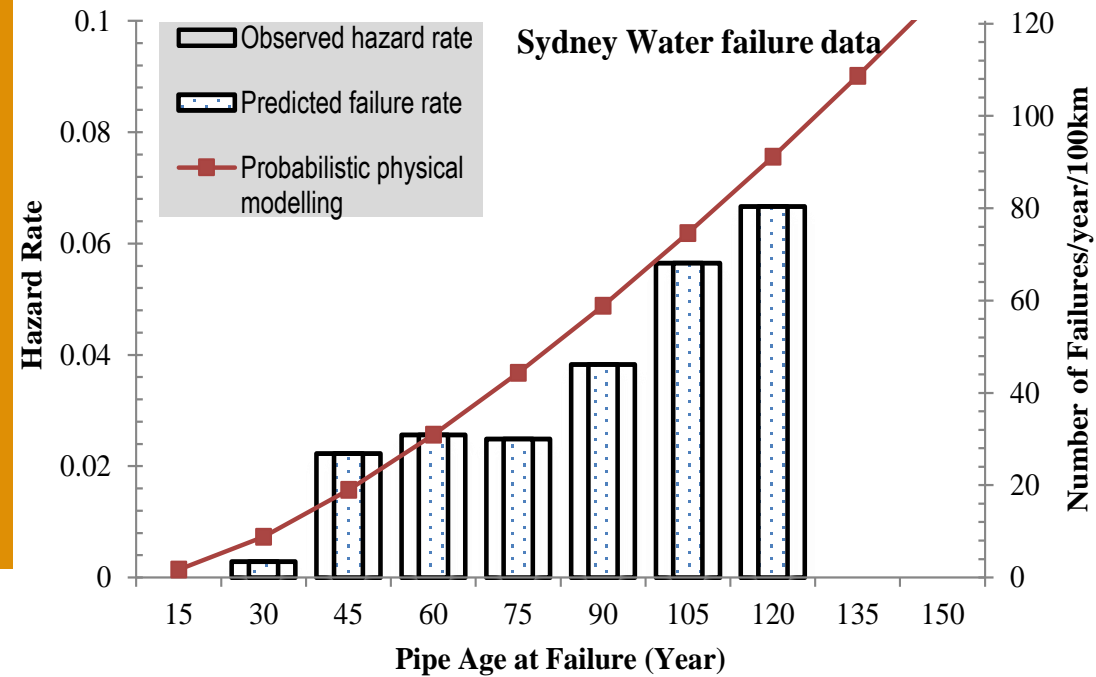
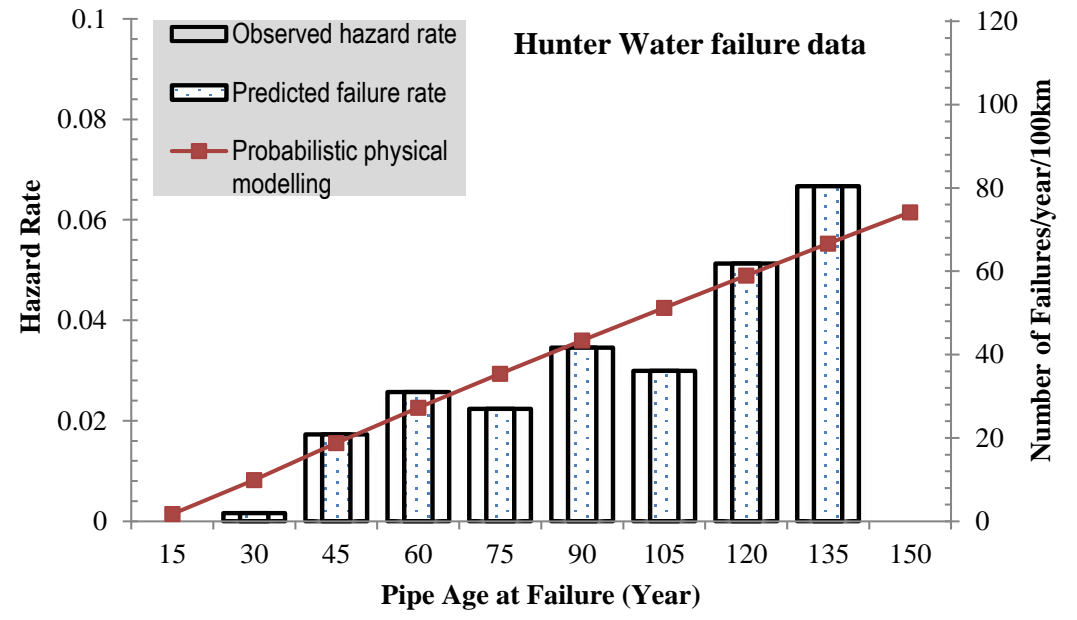
The **failure rate**, in terms of number of failures per 100 km per year, is commonly recognized by water utilities as a basis for failure prediction for their assets. The hazard rate as obtained from probabilistic physical modelling as well as the statistical failure data analysis can be easily converted into this practically conceptual failure rate. Assuming statistical independence between pipes' failure, the following relationship holds

$$Q(T) = H(T) \times N_T$$

where  **$Q(T)$**  denotes failure rate, in unit of number of failures per 100 km per year,  **$H(T)$**  is hazard rate for a typical pipe (section), and  **$N_T$**  is the number of pipes (sections) for a 100 km cohort in a particular year  **$T$** .

➤ Hunter Water failure data contain a total of 832 failures of cast iron pipes, lengths varying from several meters up to 500 m, and some long pipes can experience multiple failures after repair. On average, each failure can happen on a typical length of 83 m. As a result, it is deemed that a 100 km cohort will roughly consist of  $N_T = 1205$  such typically long pipes that are under the risk of failure. This, in turn, produce the predicted failure rate  $Q(T)$ , as shown in Fig. 14(a) by the second vertical axis.

➤ Similarly, the predicted failure rate  $Q(T)$  for Sydney Water is shown in Fig. 14(b).



# Summaries and Conclusions

- Physical modelling provides a good understanding of the mechanics of pipe behaviour. It can reveal the relationship between pipe failure and relevant physical properties.
- As the basis of risk assessment and strategy planning, the lifetime failure probability can be obtained from physical modelling, by taking all uncertain factors into account.
- The hazard rate obtained from physical modelling agrees well with the observed failure data, and updating of some key physical parameters is possible by such comparative studies, more detailed works are ongoing.
- The spatial variation of corrosion pits, as observed from condition assessment, was not considered yet, and shall be included in the future work.