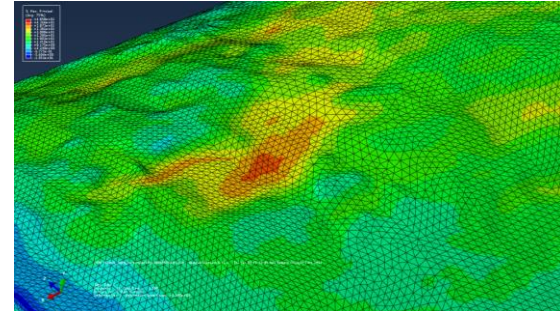
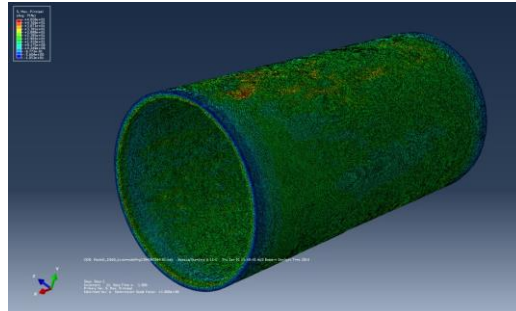




Pipe Progress Presentation Activity 4e

Prof. Jayantha Kodikara
Dr. Jian Ji

18th Feb 2016



Contents

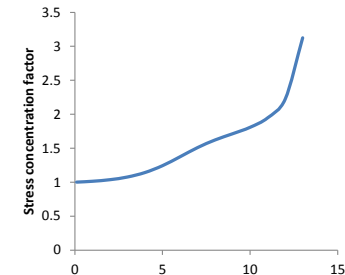
- Updating of probabilistic mechanical model for stress analysis
- Variation of physical parameters
- Back-figuring of physical parameters using Bayesian inverse analysis
- Probabilistic modelling for residual lifetime prediction
- Staged Summaries

Understanding of pipe failure mechanism

- Pipe failure involves complicated mechanics, for long-term failure prediction, we use the concept of *the leak before failure*, ignoring the small time lag between leakage and pipe burst.
- Corrosion patch causes localized stress increase: **FEM analysis**
- Limit state of *the leak before failure* : max stress → tensile strength

$$g(\mathbf{x}, t) = [\sigma_y - \sigma_{nom}(\mathbf{x}, t) \cdot SCF(t)] / \sigma_y$$

Pipe performance will deteriorate over time



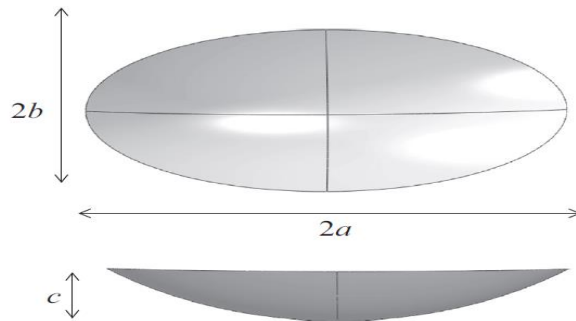
- Evidence: for observed failure, the stress reaches its limit state:

$$g(\mathbf{x}, t) = 0$$



Updating of SCF model for PPM

- Define the 3-parameter elliptical corrosion patch

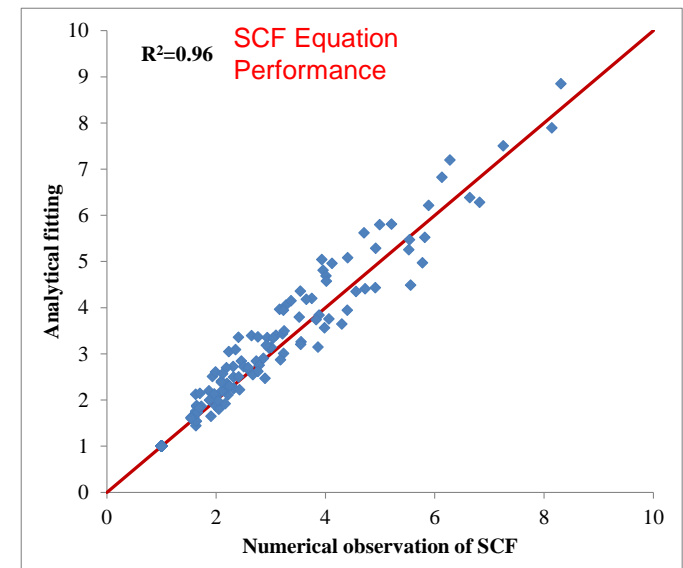


Corrosion Patch configurations:

- ✓ 2a - major axis
- ✓ 2b - minor axis
- ✓ c - depth

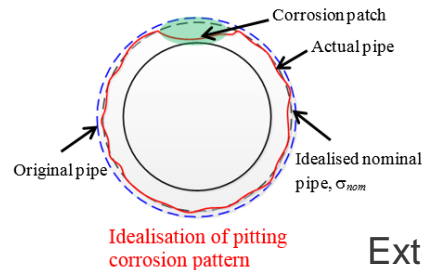
- SCF regression formula

$$SCF = 1 + \frac{\sqrt[4]{3(1-\nu^2)}}{2} \left\{ \frac{\alpha_1 \left(\frac{a}{\sqrt{RT}} \right)^{\beta_1} + \alpha_2 \left(\frac{b}{\sqrt{RT}} \right)^{\beta_2} + \alpha_3 \left(\frac{c}{\sqrt{RT}} \right)^{\beta_3}}{\alpha_4 \left(\frac{a}{\sqrt{RT}} \right)^{\beta_4} + \alpha_5 \left(\frac{b}{\sqrt{RT}} \right)^{\beta_5} + \alpha_6 \left(\frac{c}{\sqrt{RT}} \right)^{\beta_6}} \right\} \alpha_7 \left(\frac{c}{T-c} \right)^{\beta_7}$$



Some details of realistic consideration

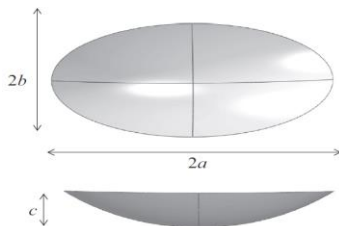
1. Coupling the uniform corrosion with patch corrosion



The uniform loss = 1/10 of the **most critical pit depth**.

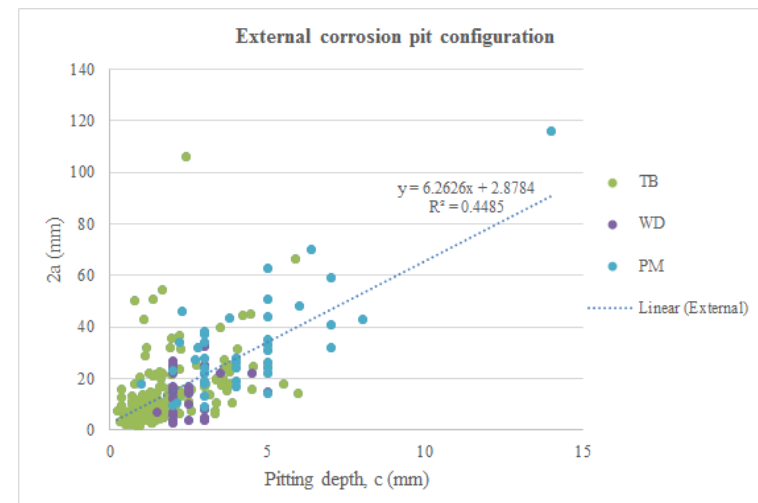
External corrosion pit configuration (from Jerry's technical report)

2. Patch configuration



Expected configuration:
 $2a = 6.262c + 2.8784$

Expected aspect ratio:
 $a/b = 1.15$



Some details of realistic consideration

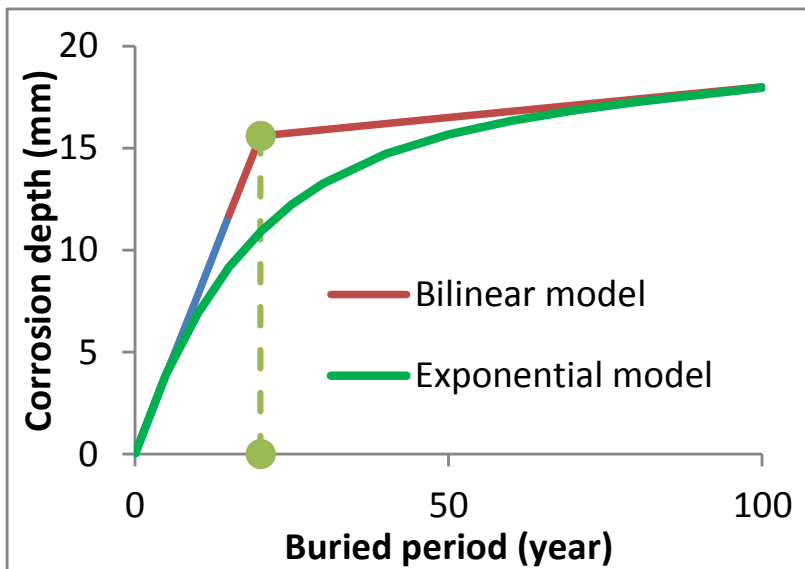
3. Pitting corrosion model

- Bilinear model: r_s (mm/y), c_s , T_0
- Exponential model: a , k , $c=1/T_0$

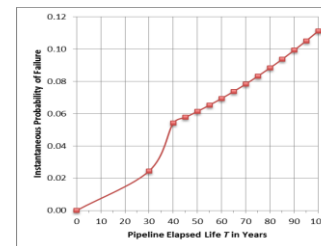
Pit depth: $d = at + k(1 - e^{-ct})$

Long-term rate: a (mm/y)

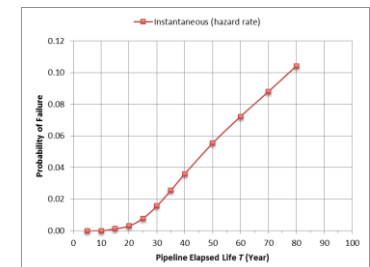
Initial corrosion rate: $a+kc$ (mm/y)



- Two models are equivalent to describe the long-term corrosion behavior
- Exponential model is more versatile, better for numerical simulation, avoiding the breaking point

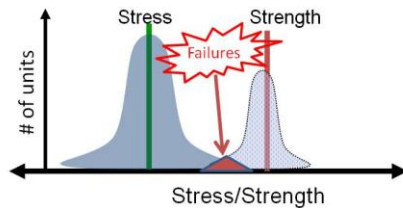


PPM by bilinear model



PPM by exponential model

Probabilistic modelling



Stress limit state

Tensile strength



Operating pressure



Pipe diameter

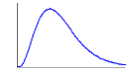


Wall thickness

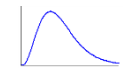


SCF(t)

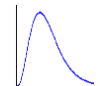
Patch config a



Patch config b



Patch depth c

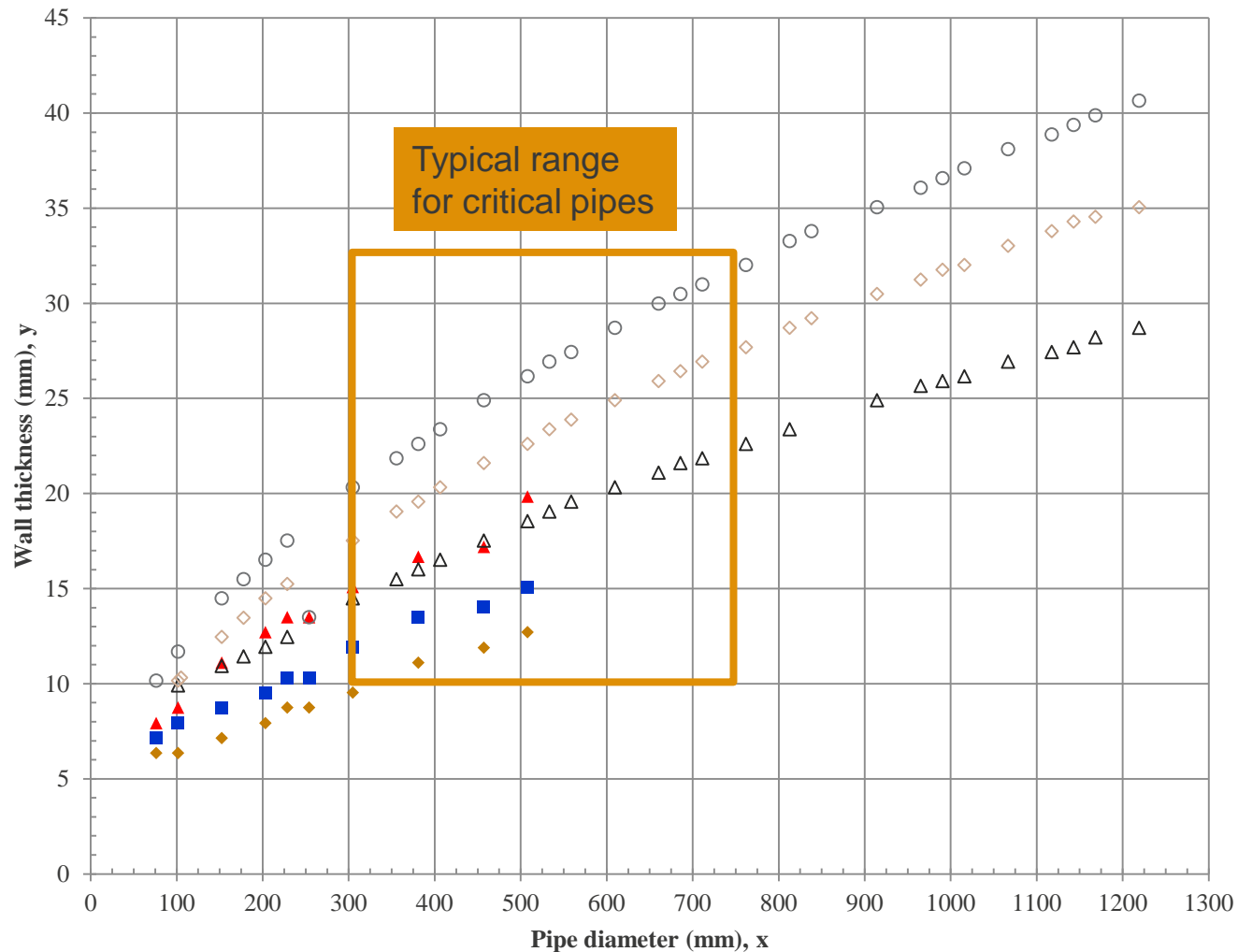


Red: time independent
Blue: time dependent

At a point of time, probability of pipe failure:

$$P_f\{g(\mathbf{x}, t) < 0\} = P\{\sigma_y - \sigma_{nom}(\mathbf{x}, t) \cdot SCF(t) < 0\}$$

Classification: diameter & wall thickness

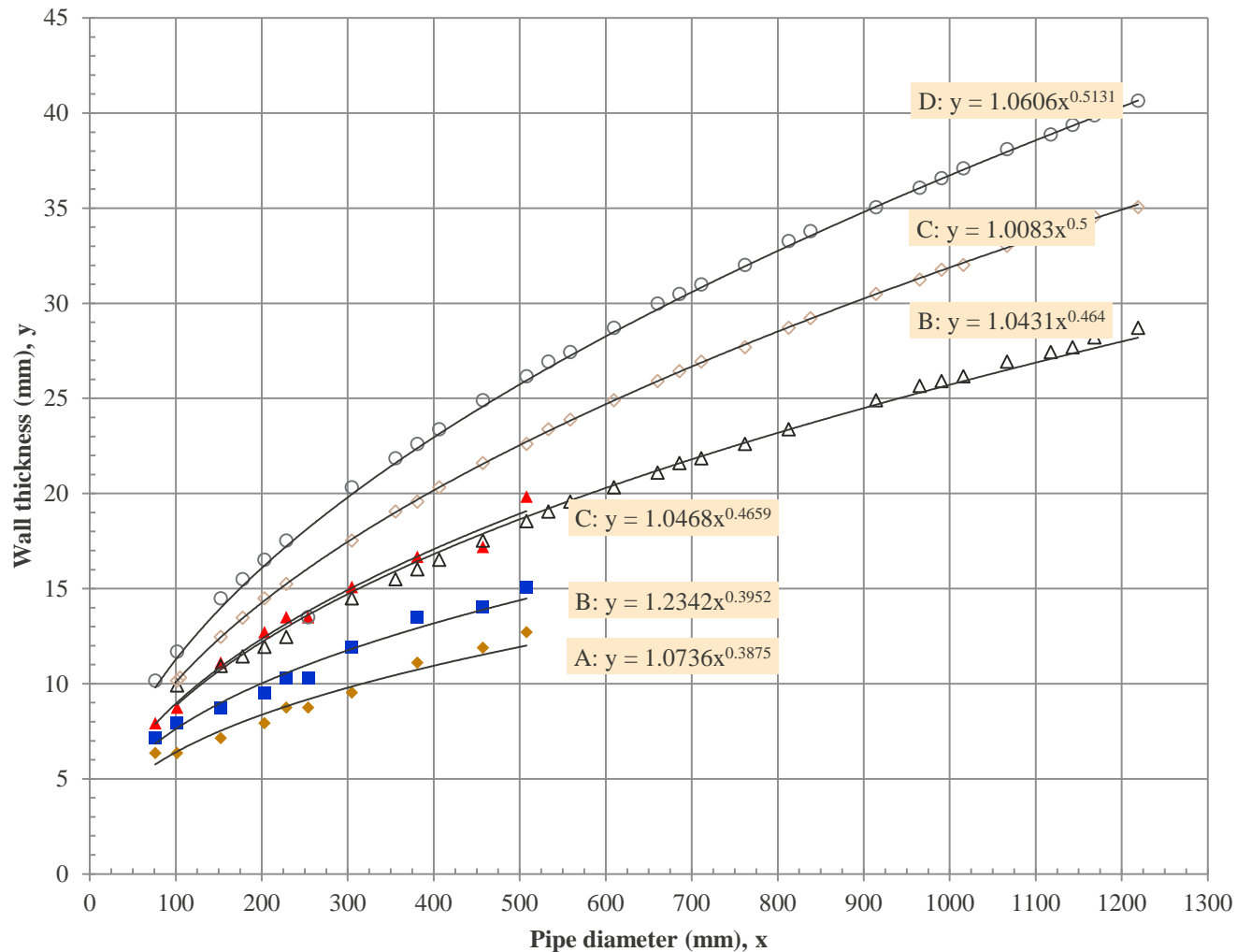


Spun cast iron pipe
(AIS, 1941)

Pit cast iron pipe
(British, 1938)

- ◆ Spun CI Class A
- Spun CI Class B
- ▲ Spun CI Class C
- △ Pit CI Class B
- ◇ Pit CI Class C
- Pit CI Class D

Quantify wall thickness & pipe diameter



Power equations are capable to quantify between wall thickness and pipe diameter

- ◆ Spun CI Class A
- Spun CI Class B
- ▲ Spun CI Class C
- △ Pit CI Class B
- ◇ Pit CI Class C
- Pit CI Class D

Contents

- Updating of probabilistic mechanical model for stress analysis
- **Variation of physical parameters (a priori consideration)**
- Back-figure of physical parameters using Bayesian inverse analysis
- Probabilistic modelling for residual lifetime prediction
- Staged Summaries

Variation of physical parameters


- For analysis of **individual pipes**:
 - Water pressure: mean value varies from 500 kPa to 1100 kPa, coefficient of variation is 0.15
 - Tensile strength
 - Australia large diameter CI pipes: mean value is 100 MPa for pit cast, and 130 for spun cast materials. Both are subject to coefficient of variation of 0.15
 - For overseas: average tensile strength seems to be higher by 30 to 50%
 - To fill the gap, we need to conduct more experiments to investigate the Australia CI pipe tensile strength
 - Wall thickness and pipe diameter: mean value depends on the diameter, coefficient of variation is 0.1

Variation of physical parameters

- For analysis of **pipe cohorts** in the network:
 - Water pressure: mean value varies from 500 kPa to 1100 kPa, coefficient of variation is 0.25
 - Australia large diameter CI pipes Tensile strength: mean value is 100 MPa for pit cast, and 130 for spun cast materials. Both are subject to coefficient of variation of 0.4
 - Pipe diameter: mean value depends on the cohort, coefficient of variation is 0.1
 - Wall thickness: mean value depends on the diameter, coefficient of variation is 0.25

Contents

- Updating of probabilistic mechanical model for stress analysis
- Variation of physical parameters (a priori consideration)
- **Back-figuring of physical parameters using Bayesian inverse analysis**
- Probabilistic modelling for residual lifetime prediction
- Staged Summaries

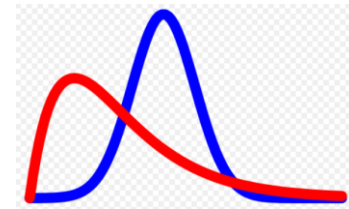


**Back Figure Physical Parameters
Using
Bayesian Approach**

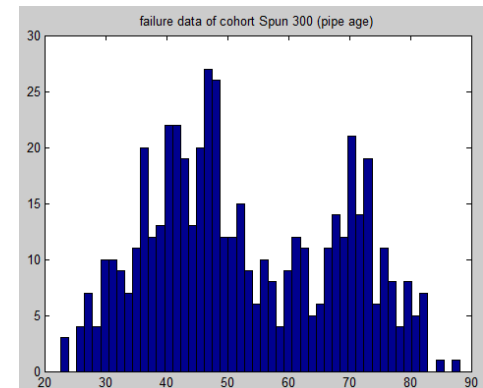
**A systematic analysis
to combine
our physical models
with
observed failure data**

Basic physical parameters and prior knowledge

- Physical parameters with uncertainty
 - Water pressure YES
 - Corrosion rate YES
 - CI tensile strength YES
 - Wall thickness, cohorted
 - Patch lateral corrosion size YES
- Objective: Use observed pipe failures to update our knowledge of the a priori assumption of some important physical parameters (uncertainty)



Histogram of failure lifetime record

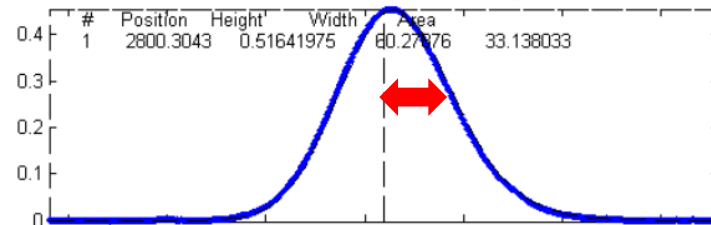
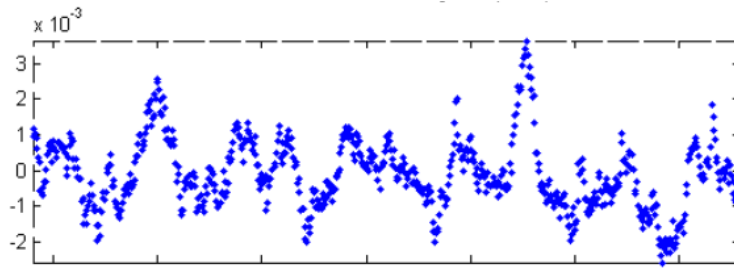


Physical model error in pipe failure prediction

- Mechanical model M that can perfectly predict the response to inputs at time t .

$$M = g(\mathbf{x}, t) = [\sigma_y - \sigma_{nom}(\mathbf{x}, t) \cdot SCF(t)] / \sigma_y$$

- Imperfection and model error



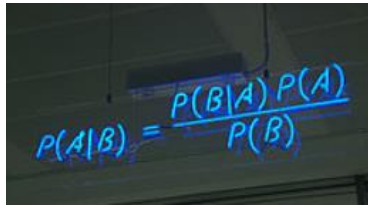
Assumption:

SCF model and stress model are good enough, with prediction error for $LSS = 0.2$, i.e., 70% prediction lies between 130 ± 26

Bayesian inverse analysis

- Bayes' Theorem

A simple introduction in Wikipedia


$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

- Bayes' Theorem: the posterior PDF depends on the prior PDF and likelihood

$$f_{\mathbf{X}}(\tilde{\mathbf{x}} | y_j) \equiv f_{\mathbf{X}}(\tilde{\mathbf{x}}) = \underbrace{c}_{\text{Not known}} p_{\mathbf{X}}(\tilde{\mathbf{x}}) L(y_j | \tilde{\mathbf{x}})$$

Computational tricks: Markov Chain Monte Carlo (MCMC) simulation

where the Likelihood function for indirect Bayesian updating

$$L(y_1, \dots, y_N | \tilde{\mathbf{x}}) \propto \prod_{j=1}^N \frac{1}{\sigma} \exp \left[-\frac{1}{2} \frac{(y_j - \mathcal{M}(\tilde{\mathbf{x}}, t_j))^2}{\sigma^2} \right]$$

MCMC algorithm

■ Metropolis-Hastings Algorithm

A proposal distribution, like Normal

$$x_{i+1} = \begin{cases} \tilde{x} \sim q(x | x_i) & \text{with probability } \alpha(x_i, \tilde{x}) \\ x_i & \text{else} \end{cases}$$

Acceptance probability

$$\alpha(x_i, \tilde{x}) = \min \left\{ 1, \frac{f_X(\tilde{x}) q(x_i | \tilde{x})}{f_X(x_i) q(\tilde{x} | x_i)} \right\}$$
$$\alpha(x_i, \tilde{x}) = \min \left\{ 1, \frac{f_X(\tilde{x})}{f_X(x_i)} \right\}$$

■ High-dimensional MCMC sampler

- Metropolis within Gibbs: sampling variables one by one, each with a metropolis trick.
- Casacaded Metropolis Sampling: first examine the prior acceptance ratio, then examine the likelihood acceptance ratio (Tarantola 2005).

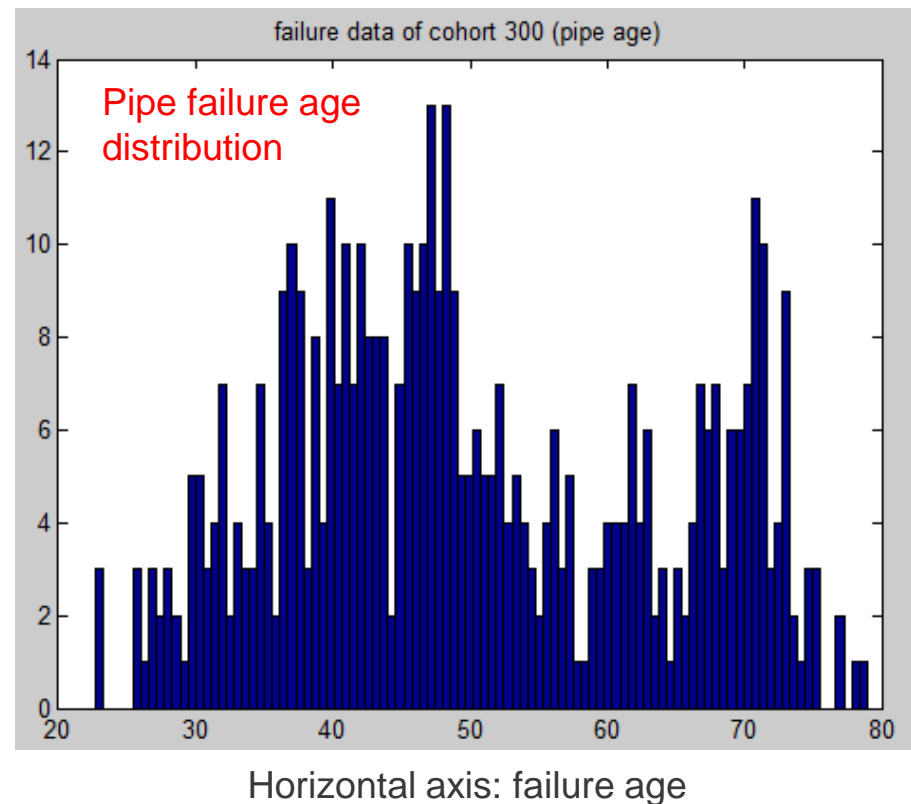


MCMC Simulation Results

- Assumption:
 - SCF model and stress model are good enough, with prediction error for LSS = 0.2, i.e., 70% prediction lies between 130 ± 26
 - Assume the time to first failure of each pipe segment.

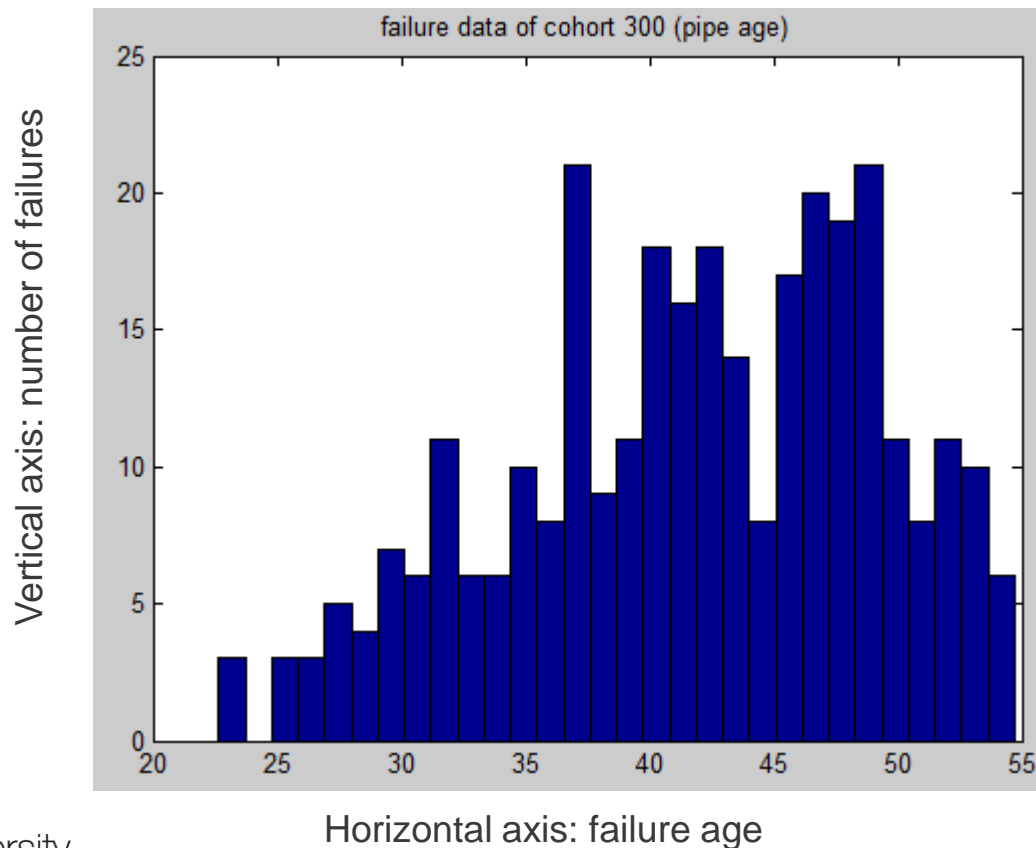
SW Failure data: Cohort 300 Cast Iron

- There are 520 (474) failure pipes for the cohort 300mm, recorded between 2001 to 2012
- Multiple failure mode shown by the data.
- Limitation: data is not fully recorded from the beginning.
- Records of early failure data shall be necessary

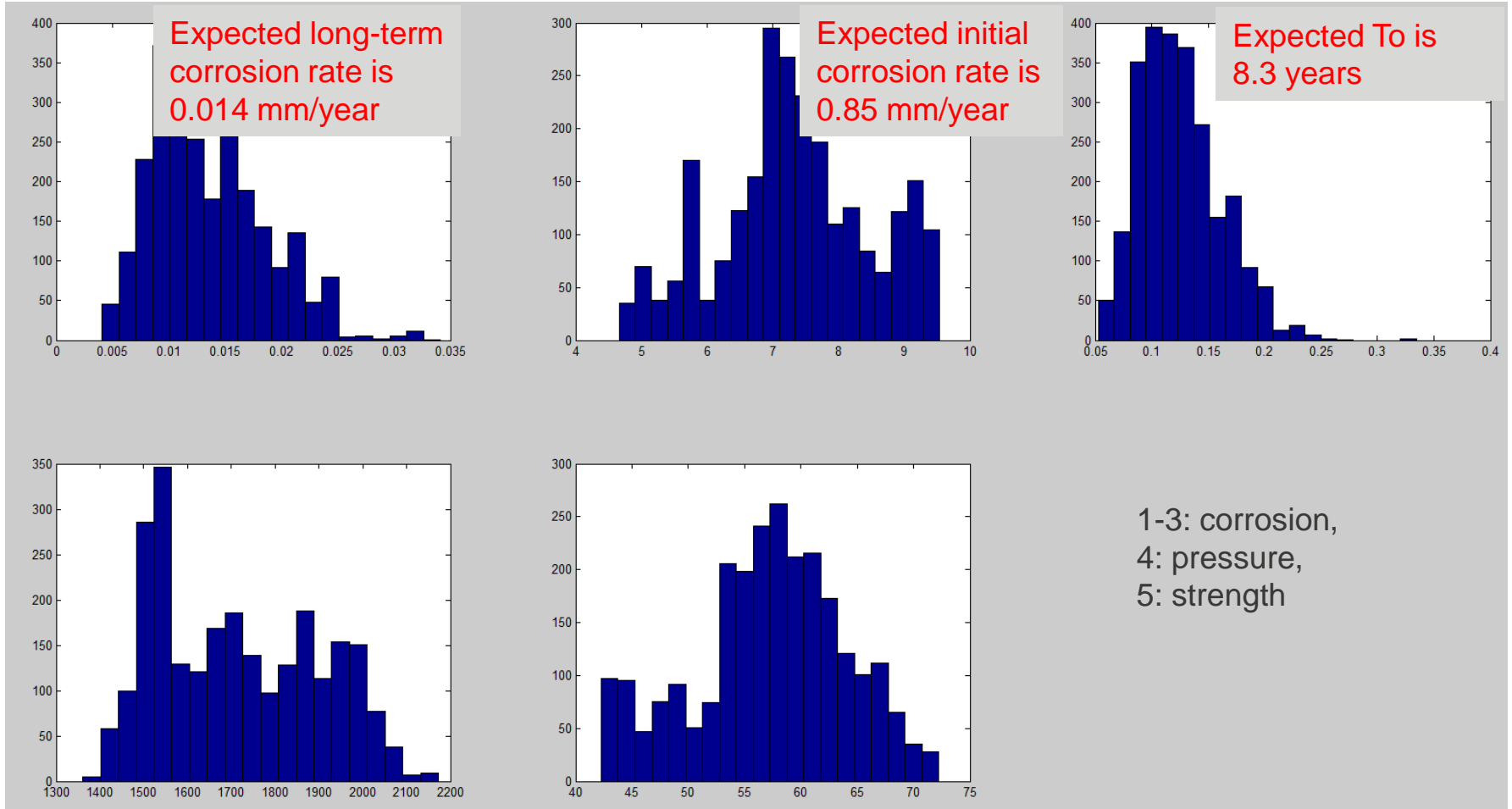


After 1930

Cohort 300 Spun CI lifetime histogram comprising 310 failure data

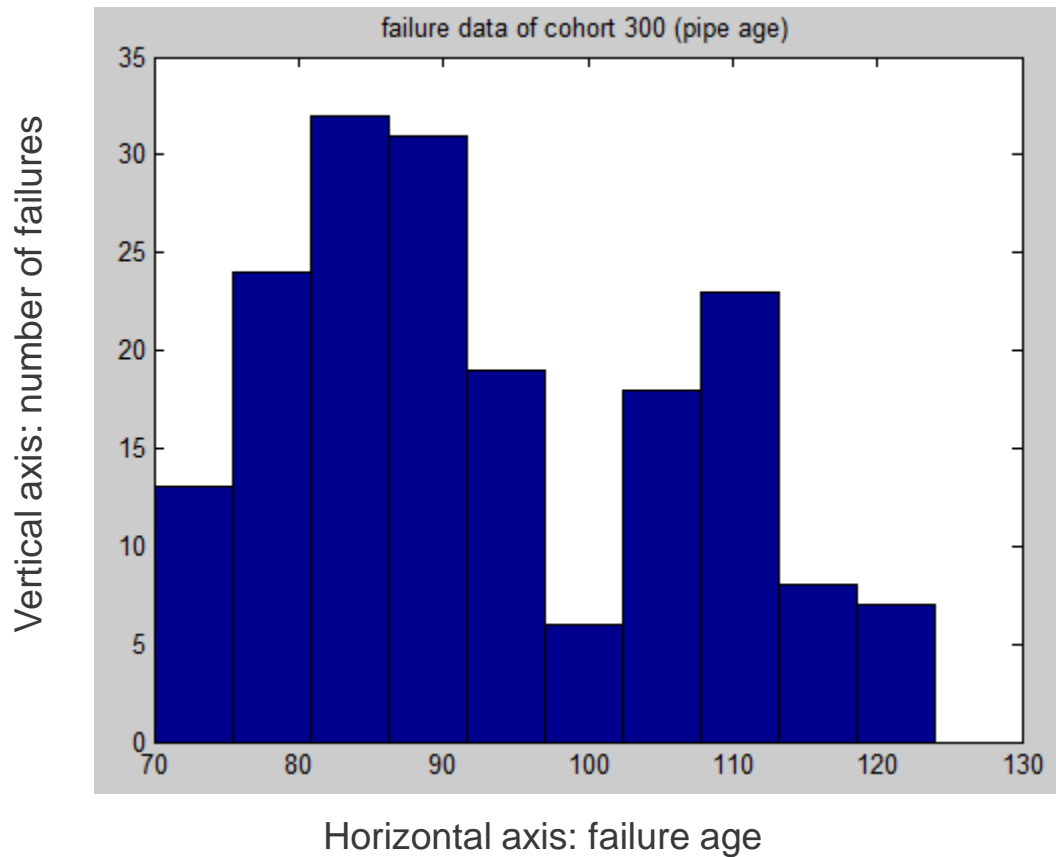


MCMC simulation results for 300 Spun CI



Before 1930

Cohort 300 Pit CI histogram 181 data



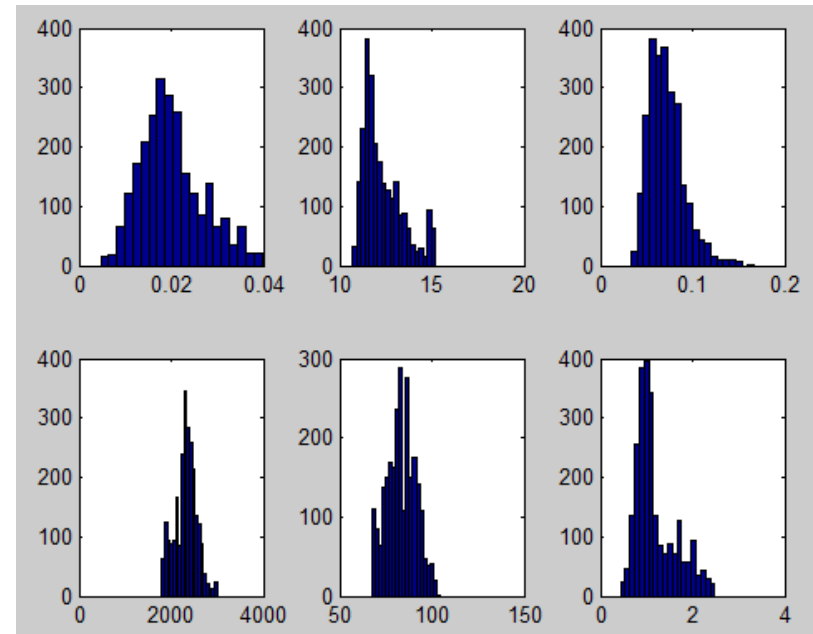
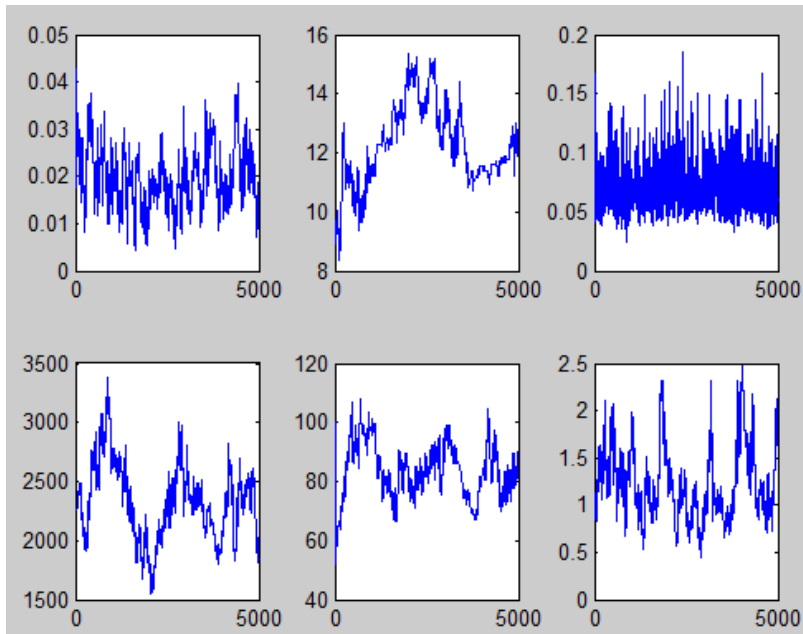
MCMC simulation results for 300 pit CI

1-3: corrosion,
4: pressure,
5: strength
6: patch size
coefficient

Expected long-term corrosion rate is 0.02 mm/year, COV = 0.35

Expected initial corrosion rate is 0.9 mm/year

Expected To is 14.5 years

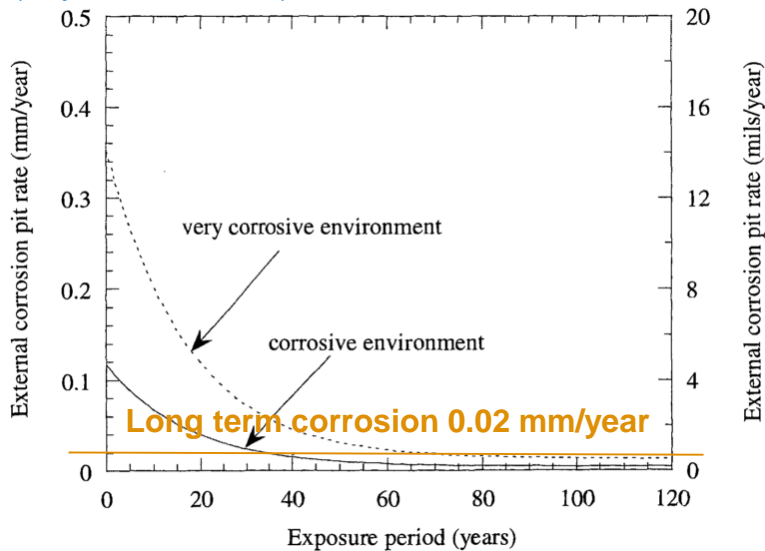


Comparison between prior and posterior mean values

- Prior, assumptions
 - Corrosion
 - $r_s = 0.05$ to 0.12 mm/year
 - Water pressure
 - 500 to 1000 kPa
 - Tensile strength
 - 100 to 130 MPa
- Posterior, back figured
 - Corrosion
 - $r_s = 0.01$ to 0.02 mm/year
 - Water pressure
 - 1000 to 1500 kPa
 - Tensile strength
 - 70 to 100 MPa

Typical long-term corrosion rates for ferrous pipes

Long-term corrosion rate for grey cast iron pipes
(Rajani et al., 2000)



In this study, the back figured rs is between 0.01 to 0.02 mm/year, indicating very low to low long-term corrosion.

Effect of early stage of corrosion is not clear because pipe failures were not recorded, for example, before 1980's

Need more data for calibration

Long-term corrosion rate for ductile iron pipes, reported in North America (Rajani et al., 2011)

Soil corrosivity	Corrosion parameters for two-phase model			
	Minimum corrosion rate, a (mm/year)	k (mm)	c (1/year)	Maximum corrosion rate, $a + kc$ (mm/year)
Very low (VLC)	0.0042	1.95	0.058	0.1173
Low (LC)	0.0210	9.75	0.058	0.5865
Moderate (MC)	0.0252	11.70	0.058	0.7038
High (HC)	0.0294	13.65	0.058	0.8211
Very high (VHC)	0.0336	15.60	0.058	0.9384

REFERENCE:

- Rajani, B. Investigation of grey cast iron water mains to develop a methodology for estimating service life. American Water Works Association. 2000.
- Rajani, B., Kleiner, Y., Krys, D., Scientific, C. Long-term Performance of Ductile Iron Pipes. Water Research Foundation. 2011.

Contents

- Updating of probabilistic mechanical model for stress analysis
- Variation of physical parameters (a priori consideration)
- Back-figure of physical parameters using Bayesian inverse analysis
- **Probabilistic modelling for remaining life prediction**
- Staged Summaries

Updating of PPM prediction

- PPM
 - At discretization point of time T, the failure mechanism is probabilistically modelled. The *instantaneous failure probability* is derived, $P_f(T)$.
 - Also called hazard rate $h(t) = P_f(t)$.

- Reliability Function, Cumulative Distributions
 - CDF defined by $F(T) = P_f(t < T)$
 - *Reliability function* is the survival probability over lifetime interval 0 to T, $R(0, T) = 1 - P_f(t < T)$

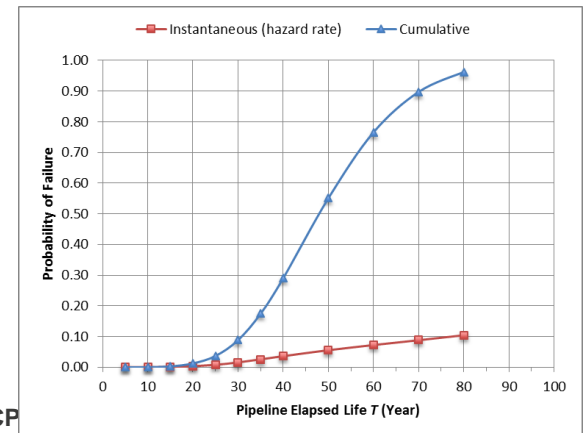
- Mathematical relationships (yearly)
 - $h(t) = f(t) / [1 - F(t)]$
 - $F(t_{i+1}) = F(t_i) + [1 - F(t_i)] P_f(t_i)$

Fig below: Cohort 300 mm CI prediction based on updated physical parameters

SW

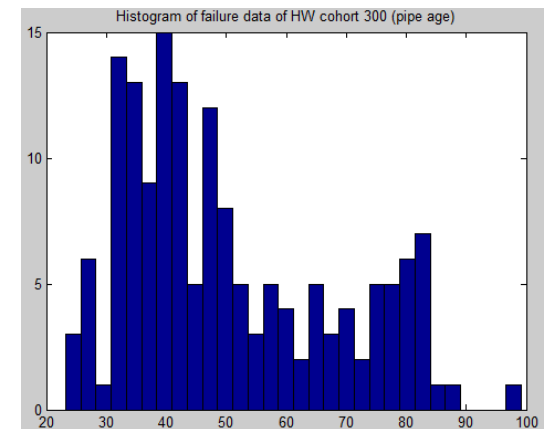
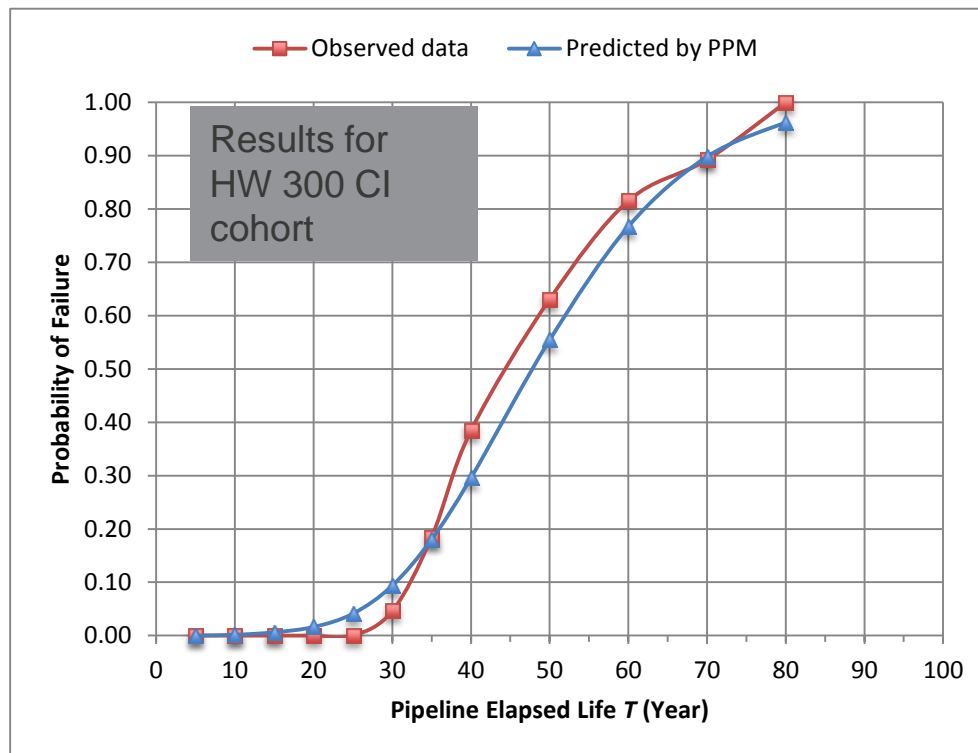


HW



CP

Comparison between PPM and Data



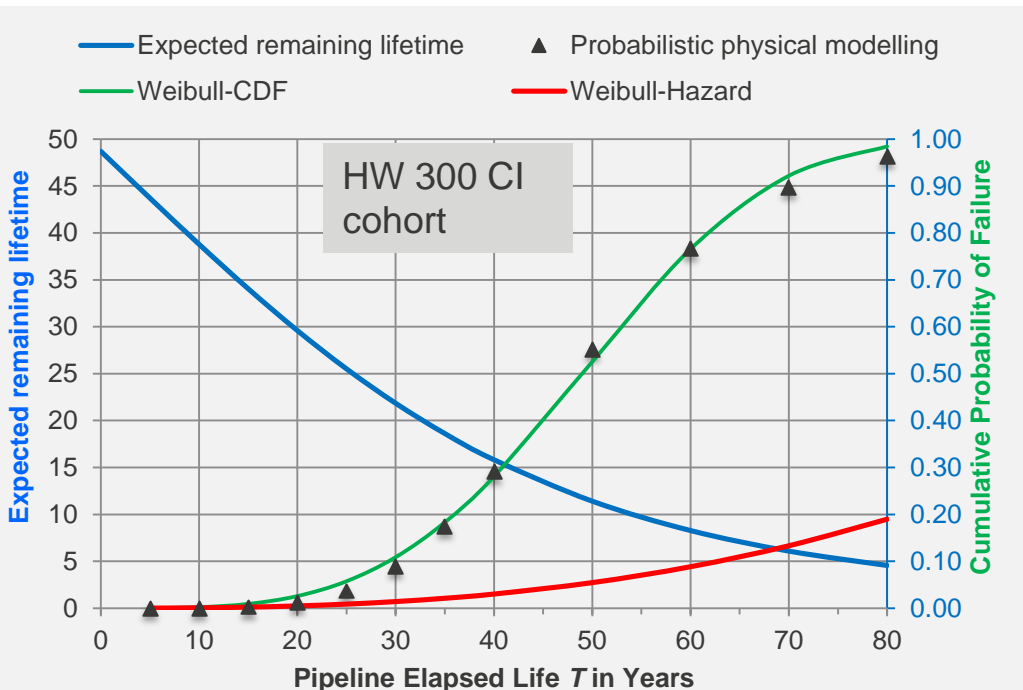
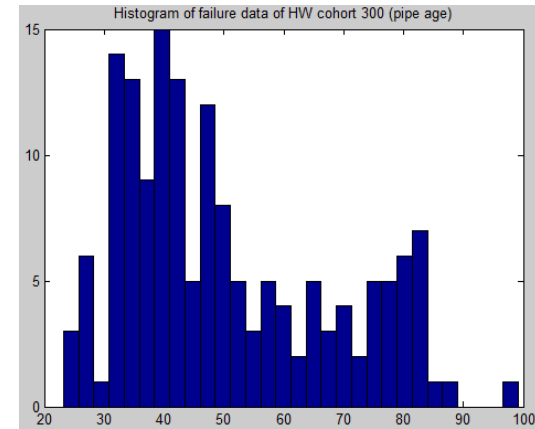
- ✓ The modelling results based on back-figured uncertainties are comparable with failure data observations
- ✓ More data needed to minimize the discrepancy

Expected remaining lifetime to failure

Suppose a pipe has survived to T , what is the expected remaining life?

Definition: $\mu(T_r) = E\{t - T_r | t > T_r\} = \left[\int_{T_r}^{\infty} [1 - F(x)] dx \right] / [1 - F(T_r)]$

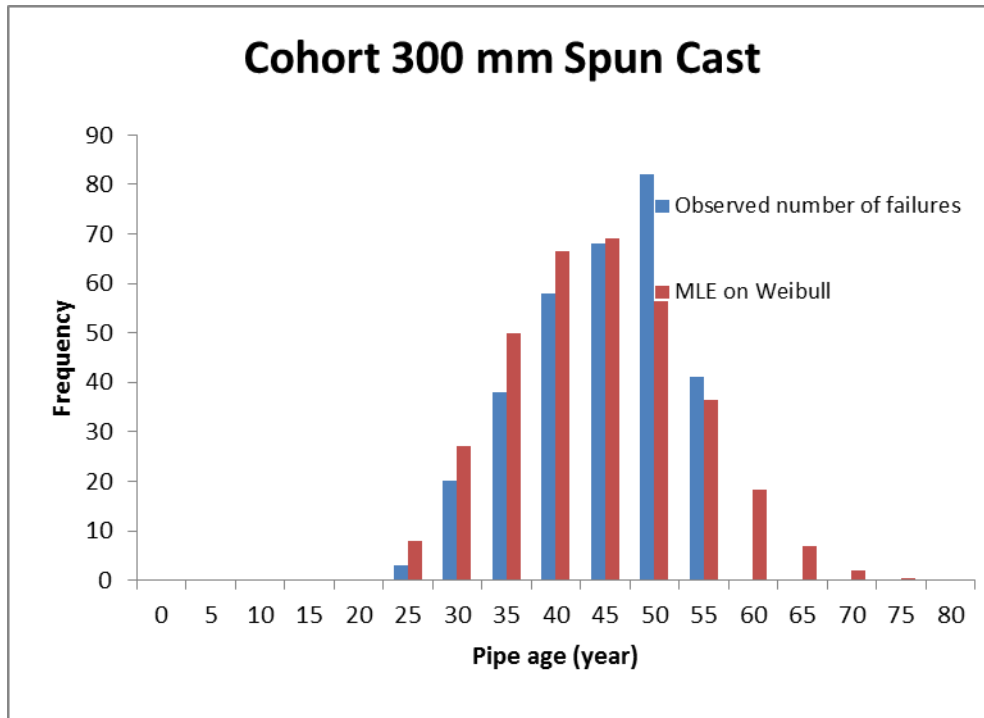
For Weibull: $\mu(T_r : \alpha, \beta) = \left[\int_{T_r}^{\infty} e^{-(x-T_0/\beta)^\alpha} dx \right] / e^{-(T_r-T_0/\beta)^\alpha}, \quad \forall T_r \geq T_0$



Derivation of Weibull-based Hazard Function

T_0	0.0232	Location parameter
alfa	3.65	Shape parameter
beta	54.17	Scale parameter
R^2	0.9981	Index of fit

Statistical: Weibull fitting to failure data



Incompleteness of data

Left truncated Weibull

MLE to estimate the parameters

Contents

- Updating of probabilistic mechanical model for stress analysis
- Variation of physical parameters (a priori consideration)
- Back-figure of physical parameters using Bayesian inverse analysis
- Probabilistic modelling for residual lifetime prediction
- **Staged Summaries**

Summaries

- ✓ Elliptical corrosion patch configurations were incorporated to update the PPM modelling work
- ✓ Based on the mechanical models (stress, corrosion patch SCF, etc.), the LONG-TERM corrosion rate parameters are back-analyzed to conform to cohort failure data.
- ✓ Typically, r_s is around 0.01 to 0.02 mm/year, on average. This finding indicates that corrosion patches develop rapidly in the initial stage, and slowly progressed for long-term.
- ✓ The water pressure yielding leakage on the corrosion patch increases with the pipe diameter. Their mean values have shown to be more than 1100 kPa for all the cohorts analyzed. This usually refers to the maximum water pressure, i.e., steady state + transient.
- ✓ Prediction of CI pipe remaining service life is possible by PPM



Thank you!